

1. A sled starts sliding on a horizontal section of a road under the influence of constant force of 150 N directed at an angle of 60° to the horizontal. After the first 10 m covered by the sled, its kinetic energy increased by 200 J. Determine the magnitude of the friction force acting on the sled.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

2. How many organic molecules with the molecular formula $C_4H_{10}O$ do exist? Note: possible E-Z isomers and enantiomers should be counted.
3. Find the minimal possible value of $x + y$, where x and y are positive integers such that $51x - 31y = 12$.
4. Assume $\lim_{n \rightarrow \infty} a_n = A$, $b_n = |a_n - A|$. Which of the following statements are correct?
 - 1) $\{b_n\}$ is a non-increasing sequence.
 - 2) There exists a positive integer N such that $\{b_N, b_{N+1}, \dots\}$ is a non-increasing sequence.
 - 3) There exists a positive integer N such that $b_n = 0$ for any $n \geq N$.
 - 4) There exists a sequence of positive integers $k_1 < k_2 < \dots$ such that $\{b_{k_1}, b_{k_2}, \dots\}$ is a non-increasing sequence.
5. Assume that G is a graph consisting of 101 nodes and 100 edges. Which of the following could be true? (Multiple choice)
 - 1) G contains some cycles
 - 2) G doesn't contain a cycle
 - 3) G is a tree
 - 4) G is a disconnected graph
6. Given a sufficient amount of iodine is available, how many milliliters of 5% sodium hydroxide solution is needed to fully convert 7.4g of butan-2-ol to sodium propanoate, sodium iodide, water and iodoform? Take integer atomic masses. Give the integer answer without units.
7. You have a string. You are allowed to perform two types of operations. 1. Add two equal characters to some position. Examples: AA \rightarrow ABBA, AA \rightarrow CCAA.
2. Delete a substring in the form XYX where X and Y are different characters. Examples: BABA \rightarrow A, BABA \rightarrow B, you cannot delete CCC, because characters must be different.
What is the minimal number of such operations required to turn the string "ABC" into the empty string?
8. A triangular numeric table consists of four rows. The first row contains four arbitrary integers. The second row contains three sums of the pairs of the adjacent numbers from the first row (in the respective order), the third row contains two sums of the pairs of the adjacent numbers from the second row, and the fourth row contains the sum of the two numbers from the third row. What is the maximal number of odd numbers among these ten integers?

9. A given solution ($\text{pH} \approx 7$) contains several ions with the concentrations listed in the table.

Ion	Concentration (mM)
Potassium K^+	22
Magnesium Mg^{2+}	44
Sodium Na^+	66
Carbonate CO_3^{2-}	44
Nitrate NO_3^-	66
Chloride Cl^-	?

What is the concentration of chloride ions in the solution (in mM)? Give the integer answer without units.

10. Which of the following is true? (Multiple choice)
- 1) The DNS used by OS allows a computer to identify other computers.
 - 2) In order to connect to the internet, each computer needs a unique numerical code, which is known as IP address.
 - 3) Internet Protocol version 4 (IPv4) defines an IP address as a 32-bit number, but IPv6 defines an IP address as a 64-bit number.
11. Three balls of radii r_1 , r_2 , and r_3 are placed on a flat table such that every two balls are touching each other. They are touching the table in the points A_1 , A_2 , and A_3 , respectively. Given $A_1A_2 = 4$, $A_2A_3 = 6$, and $A_1A_3 = 8$, find r_2 .
12. With respect to each other, D-galactose and sucrose are
- 1) structural isomers
 - 2) epimers
 - 3) enantiomers
 - 4) diastereomers
 - 5) none of the above
13. The speed of a motorboat in still water is 15 km/h. The boat went 30 km downstream and then came back (upstream) in a total of 4 hours 30 minutes. What is the speed of the stream?
14. The conductors of a power line have an effective resistance of $R = 200$ Ohm. Electric power of $P = 40$ MW is transmitted through the power line. How many megawatts (MW) of thermal losses are released due to the heating of the power line? The voltage of the power line is $U = 400$ kV.
- The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.
15. Using only your fingers as digits of a binary number, how far can you count with both of your hands? The answer must contain the maximum integer that you can represent, presented in decimal notation.
16. A 5 kg ball is thrown down from a 10 m height with a speed of 6 m/s. The ball bounces back half of its initial height. How much energy did the ball lose due to the impact with the ground? Air resistance is negligible.
- The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.
17. In triangle ABC , $\angle CAB = 19^\circ$ and $\angle BCA = 22^\circ$. E is a point on CA such that $\angle CEB = 60^\circ$ and F is a point on AB such that $\angle AEF = 60^\circ$. Find $\angle BFC$. Give the answer in degrees.

18. You are given a program in Python 3. What will appear in the screen if the user inputs a string “megapolis” (without quotes)?

```
1 s = input()
2 print(s[1::]+s[:2:]+s[::3])
```

1) megapoliselis; 2) egapolismemal; 3) megapoligapolisa; 4) egapolimamal.

19. Which of the following bond angles are less than that of $H-O-H$ in H_2O ? (Multiple choice)

- 1) $H-N-H$ in NH_3
- 2) $H-S-H$ in H_2S
- 3) $H-C-H$ in CH_4
- 4) $Cl-I-Cl$ in ICl_4^-

20. Determine the output of the following program:

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 const int n = 100;
4 int answer = 0;
5 void f(int l, int r, int ql, int qr) {
6     ++answer;
7     if (qr < l || ql > r)
8         return;
9     if (ql <= l && qr >= r)
10        return;
11    int mid = (l + r) >> 1;
12    f(l, mid, ql, qr);
13    f(mid + 1, r, ql, qr);
14 }
15 int main() {
16    int l = 2, r = 33;
17    f(1, n, l, r);
18    cout << answer << endl;
19 }
```

21. Let ABC be a triangle. Line p is the tangent to the circumcircle of the triangle at point C . Line q is parallel to line AB and passes through point C . If angle CAB is 20° and angle ABC is 65° , what is the angle between the lines p and q ? (By definition, the angle between two lines is acute or right.) Give the answer in degrees.

22. Calculate the minimal height of a binary tree that contains 2019 nodes. The height of a tree is the maximum number of nodes on a path from the root to a leaf of the tree.

23. Two vertices of a square lie on a circle, the other two lie on a tangent line of this circle. Given that the radius of the circle is 5 cm, find the area of the square. Give the answer in cm^2 .

24. Andrew and Bob are given a number n . Andrew and Bob take turns, Andrew takes the first turn. On his turn a player chooses d such that d is a natural number ranging from 1 to $n-1$ and d is a divisor of n . Then d is subtracted from n . The player who can't make a move loses. For which values of n Andrew can win no matter how Bob plays? (Multiple choice)

- 1) 2; 2) 3; 3) 2018; 4) 2019; 5) 2020.

25. Point K lies on the hypotenuse AB of a right triangle ABC so that $CK = BC$. The bisector of angle BAC intersects BC at point L . Given that CK cuts segment AL in half, find angle ABC . Give the answer in degrees.

26. The simple gravity pendulum is 1.5 meters long. What is the frequency of the pendulum oscillations in millihertz (mHz)?

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

27. 2.00 g of solid sodium hydroxide are added to 200 mL of acetic acid with pH 2.38. What would be the pH value of the solution obtained? The acidity constant of acetic acid is $1.75 \cdot 10^{-5}$.

1) 1.88 2) 2.87 3) 3.48 4) 4.28 5) 12.20

28. The area S of a rectangle and its perimeter P satisfy the following equation:

$$S = \frac{P^2}{6} - 3P + 12.$$

What is the square of the maximum possible length of a diagonal of such rectangle?

29. A ball is attached to a string and moves along a circular path in a horizontal plane. The string makes an angle of 30° to the vertical axis and has a 50 cm length. Then it starts to move faster and the angle increases. Determine the angle (in degrees) between the vertical axis and the string if it is turning at a frequency two times larger.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

30. Let a , b , c and d be real numbers satisfying

$$\begin{cases} (a+b)(c+d) = 13, \\ (a+c)(b+d) = 24, \\ (a+d)(b+c) = 25. \end{cases}$$

Find the smallest possible value of $a^2 + b^2 + c^2 + d^2$.

31. Find the number of undirected connected graphs composed of four labeled nodes. For example, there are 4 connected graphs of three labeled nodes.

32. Consider an arithmetic progression a_1, a_2, \dots such that $a_4 + a_8 + a_{12} + a_{16} = 224$. Find the sum $a_1 + a_2 + a_3 + \dots + a_{19}$.

33. A sample of gaseous xenon fluoride XeF_n was confined in a bulb at a pressure of 24 Torr. Then hydrogen was added to the bulb until the pressure reached 96 Torr. Next, xenon fluoride reacted completely as a result of electric discharge affording Xe and gaseous HF , which was accompanied by the pressure increase up to 144 Torr. Determine n .

34. A truck, which has a mass of 3750 kg, moves at a speed of 54 km/h. When it arrives to a road intersection, it crashes with a car, which is moving at 90 km/h perpendicularly to the truck. The mass of the car is 1250 kg. Determine the percentage of energy lost in the collision, considering both vehicles are moving together after they have crashed.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

35. Consider a regular square pyramid $SABCD$ (with apex S) with $AB = 21$ and $AS = 42$. Points N and K belong to the edges CD and SC , respectively, and satisfy $DN : NC = SK : KC = 2 : 5$. Plane α passes through NK and is parallel to AS . Find the square of the distance from point B to α .

36. Which of the following choices requires two bits of information to store?

1) 1, 2, 3; 2) ON/OFF; 3) 0, 1, 2, 3, 4; 4) A, B.

37. A sprinter runs 100 m in 12 s. For the first 20 m his acceleration is constant and for the remaining distance his speed is constant. Calculate sprinters speed in the second part of the race.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

38. Quick sort is a commonly used algorithm, its time complexity on average is _____, time complexity for best possible input is _____, time complexity for worst possible input is _____.

- 1) $O(n \log n)$, $O(n)$, $O(n \log n)$
- 2) $O(n \log n)$, $O(n)$, $O(n^2)$
- 3) $O(n \log n)$, $O(n \log n)$, $O(n^2)$
- 4) $O(n^2)$, $O(n \log n)$, $O(n^2)$
- 5) $O(n^2)$, $O(n^2)$, $O(n^2)$

39. Let x and y be positive real numbers. Define $a = 1 + (x/y)$ and $b = 1 + (y/x)$. Given that $a^3 + b^3 = 50$, compute $a^4 + b^4$.

40. Determine the output of the following program.

```
1 public static void main(String[] args) {
2     System.out.println(Math.max(what("180", 3), what("AB", 2)));
3 }
4 public static int what(String str, int n) {
5     if (n == 1)
6         return str.charAt(0) - '0';
7     return (10 * what(str, n - 1) + str.charAt(n - 1) - '0');
8 }
```

41. The three roots of the polynomial $x^3 - 14x^2 + Bx - 84$ are the side lengths of a right triangle. Find B .

42. A piece of shiny, golden iron mineral was placed in a beaker which contained hot concentrated solution of nitric acid. The violent reaction was accompanied by evolution of a brown gas. The latter was completely removed from the reaction mixture, and formation of a dark orange solution was observed. The post-reaction solution gave a bulky white precipitate with barium chloride. What is the sum of all stoichiometric coefficients of the first reaction written in the ionic form? (Take coefficients as minimal integers).

43. A few contestants at the IOM 2019 were wondering how long in minutes would it take for a small object to fall freely through a straight tunnel dug through the center of the Earth in order to reach its opposite side. Can you help them? Feel free to assume that the Earth has a perfect spherical shape with radius 6400 km and uniform distribution of its mass. Air resistance may be neglected.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

44. The edges of a cube were increased by 100%. By how many percent did the volume of the cube increase?

45. The gaseous mixture initially contained 0.40 mol of iodine, 0.40 mol of bromine, and 0.20 mol of iodine bromide, $I\text{Br}$. After establishment of the equilibrium $I_2 + Br_2 \rightleftharpoons 2I\text{Br}$ the mixture contained 0.60 mol of $I\text{Br}$. Determine the equilibrium constant. Give the integer answer.

46. Which expression evaluates to the greatest number? The answer must contain its value presented as a decimal number.

$$15_8 + B1C_{16} - 100110_2 \text{ or } AB8_{16} + 127_8 + 1000_2$$

47. In hydrogen atom, the electronic energy levels depend on the principal quantum number n as follows: $E_n \sim -1/n^2$. Which of the following transitions corresponds to the largest wavelength in the emission spectrum of hydrogen atom?

- 1) $n = 2 \rightarrow n = 1$
- 2) $n = 3 \rightarrow n = 2$
- 3) $n = 4 \rightarrow n = 2$
- 4) $n = 6 \rightarrow n = 3$

48. You are given an infinite chessboard. Each cell of this board corresponds to a pair of integer coordinates. What is the minimum number of moves a chess knight needs to get from the cell (2, 9) to the cell (20, 19)? (Each time the chess knight moves to a square that is two squares away horizontally and one square away vertically, or two squares away vertically and one square away horizontally. The complete move therefore looks like a letter L.)

49. The solubility product K_{sp} of $Mg(OH)_2$ is $a \cdot 10^{-12}$. Find a (with one decimal place), if pH of the saturated solution of magnesium hydroxide is 10.4.

50. You are given a regular octahedron without walls — only edges and vertices. Each edge is made of a conductor of same electric resistance equal to 12 ohm. Find the resistance between two nearest vertices.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

51. At 323 K, the vapor pressure of 200 g of ethanol containing 23.0 g of a nonvolatile solute is $2.76 \cdot 10^4 \text{ Pa}$. At the same temperature, the vapor pressure of pure ethanol is $2.93 \cdot 10^4 \text{ Pa}$. Assuming the ideal solution, find the molecular mass of the solute (in g/mol). Round the answer to the nearest integer and give it without units.

52. In year 2001 on October 2, the date in MMDDYYYY format was a palindrome (same forwards as backwards), 10/02/2001, i.e. "10022001". What year had the last palindrome date before 10/02/2001?

53. In a water solution of KOH , the number of non-metal atoms is 26 times higher than that of metal atoms. Calculate the dilution factor affording the solution with the number of non-metal atoms 86 times higher than that of metal atoms. $M_r(H_2O) = 18$, $M_r(KOH) = 56$.

- 1) 1.6 2) 1.8 3) 2.4 4) 2.8 5) 3.3

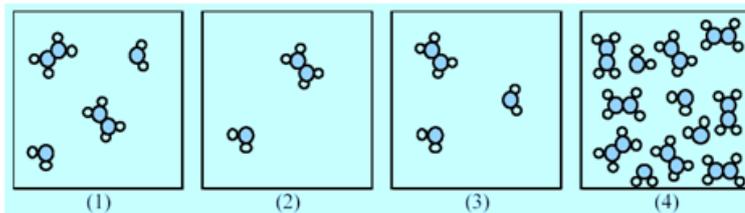
54. A polystyrene disk of radius $r = 14 \text{ cm}$ floats on the surface of the waterpool with a depth of $H = 2 \text{ m}$. A lamp is placed above the center of the disc at a height of $h = 10 \text{ cm}$. If we change the height of the lamp above the disc, we also change the size of the shadow at the bottom of the pool. What can be the maximum shade radius R ? The refractive index of water is $n = 1.5$.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

55. What is the next element in the sequence below? The answer must contain the next element of the sequence presented in octal notation.

$$11_2 \qquad 11_8 \qquad 11011_2 \qquad 51_{16} \qquad ?_8$$

56. The figure (1) below shows the equilibrium between molecules A_2B_4 and AB_2 described by the equation $A_2B_4 \rightleftharpoons 2AB_2$. What other mixtures are at equilibrium as well?



- 1) none 2) mixture (2) 3) mixture (3) 4) mixture (4) 5) all

57. A rubber string of a 75 cm length is fixed at one end to the ceiling. It is found that the string further extends by 10 cm after a small ball of mass 0.1 kg is attached to the other end of the string and reaches the equilibrium. What will be the maximum length of the string if the ball is raised to the ceiling and released from rest? Assume that when the rubber string extends, it behaves like a spring. Give your answer in centimeters.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

58. Select all the methods that can be used to synthesize $BaSO_3$ (insoluble in water):

- 1) Heating of solid $BaSO_4$ with carbon
- 2) Dissolution of H_2S in a Na_2SO_4 solution followed by addition of a few drops of $BaCl_2$
- 3) Bubbling of SO_2 through a $BaCl_2$ solution
- 4) Bubbling of SO_2 into a $Ba(OH)_2$ solution

(Multiple choice)

59. Two stones are thrown with equal speeds at different angles to horizontal, $\alpha_1 = 60^\circ$ and $\alpha_2 = 30^\circ$. What is the ratio of their maximum heights, $h_1 : h_2$?

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

60. A gaseous mixture (20 ml at STP) containing methane, argon, and oxygen was exploded. After water condensation, 12 ml of the gas mixture containing only two gases was obtained. How much oxygen (in ml) was present in the initial mixture? Give the integer answer without units.

61. Two identical balls are at rest at a given distance. The gravitational force of the left ball acting on the right ball is F . What will be the resultant gravitational force acting on the right ball (expressed in F) if the third equal ball is put in the middle between the initial two balls?

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

62. The atom of an unknown element contains $4x$ s -electrons, $6x$ p -electrons, and x d -electrons in its ground state. Determine the element. Write its atomic number in the Periodic table as the answer.

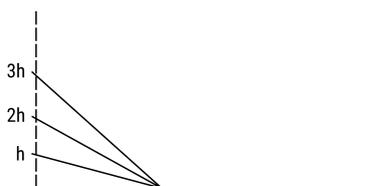
63. A charged capacitor with capacitance C is closed to an uncharged capacitance $3C$ and a resistor, which are connected in parallel. What percentage of the initial energy of the charged capacitor will be released on the resistor? The resistance of connecting wires is negligible.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

64. Let x and y be positive real numbers such that $x + y = 12$. What is the maximal possible value of x^2y ?
65. There are two integers x and y , such that the sum of these two numbers is 105. Then $\text{gcd}(x, y)$ and $\text{lcm}(x, y)$ of these two numbers are 15 and 150 respectively. What is the difference between the two numbers?

Note: $\text{gcd}(x, y)$ is the greatest common divisor of x and y , and $\text{lcm}(x, y)$ is the least common multiple of x and y .

66. A sled went down the hill three times along three different slides from the same vertical (see fig.). When the initial height of the hill was h , the sled stopped at a distance of 3 m from the base of the slide. When the initial height was $2h$, the sled stopped at a distance of 7 m. At what distance will the sled stop if it starts descending from a height of $3h$? Coefficient of friction along all paths is the same. All three slides smoothly merge into the horizontal surface, energy losses in these transition areas are considered negligible.



67. What is the minimum number of NOR (not or) gates needed to construct one XNOR (not xor) gate?
68. Find the number of non-empty subsets A of the set $\{1, \dots, 36, 37\}$ such that A has at most four elements and their sum is odd.
69. Determine at what minimum initial speed should a body of mass m be pushed so it could make a complete circle round a loop of radius 2 m located in a vertical plane. Neglect friction.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

70. We have a recursive function

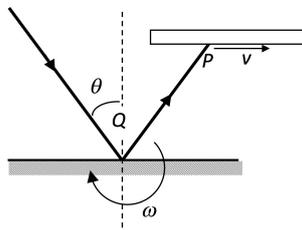
```

1  int binom(int n, int k)
2  {
3      printf("");
4      if ((k == 0) || (k == n)) return 1;
5      return binom(n - 1, k - 1) + binom(n - 1, k);
6  }
```

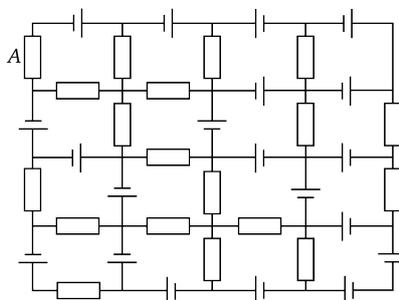
How many asterisks will be printed, if we call $\text{binom}(7,4)$?

71. Sulfur forms several binary compounds with selenium. One of these compounds was burned in air affording a solid with the mass of 333 mg and a gaseous substance with the volume of 40.3 ml (0 C, 1 atm). Determine the brutto formula of the burned compound. In the answer, give the number of selenium and sulfur atoms without spaces (for example, SeS_2 must be reported as 12).

72. A fine beam of light is incident on a plane mirror at Q . The reflected beam strikes on a screen and a light spot appears on it. The mirror is rotated in clockwise direction about Q at a constant angular speed ω . At the instant shown, $\theta = 60^\circ$ and the mirror surface is just parallel to the screen. The light spot is at P , where $PQ = 0.5$ m. At this instant, the spot appears to move across the screen at a speed of 3.0 m/s. Find ω .



73. In the diagram, each resistor has a resistance of 1 Ohm and each dry cell has an e.m.f. of 1 V and negligible internal resistance. Find the current through resistor A at the top left corner of the circuit.



74. There are two similar rings made of wire, which are isolated from each other. They share the same axis, as shown in figure 1. Now we do nothing to the right ring, but add a periodic current to the left ring. The relationship between this current and time is shown in figure 2. The direction of the arrow on the left ring is defined as positive direction. Please indicate all time periods when the interaction between these two rings manifests as attraction. Use the numbers marked in figure 2.

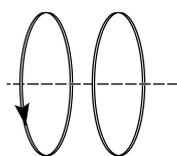


Figure 1

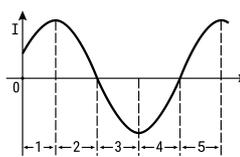
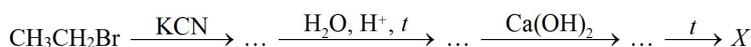


Figure 2

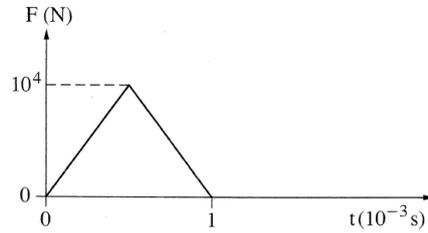
75. Suppose there are n guests at a meeting. It is known that every guest decided to present a gift to another guest in such way that every guest receives one gift. Find how many ways to present gifts are there if there are 7 guests. Two ways to present gifts are distinct if at least one guest gets a gift from a different guest.
76. Determine the molecular formula of a substance X obtained according to the following scheme:



In the answer, give the number of C, H, O atoms without spaces (for example, $\text{C}_2\text{H}_6\text{O}$ must be reported as 261).

77. Find the number of pairs of positive integers a and b such that $a < b$ and $\frac{1}{a} + \frac{1}{b} = \frac{1}{12}$.
78. The amino acid isoleucine (*Ile*) contains two stereocenters. What is the total number of the homotripeptide *Ile* – *Ile* – *Ile* stereoisomers?

79. Trolley 1 with a mass of $m_1 = 2$ kg moves along the smooth horizontal track at a velocity v_1 . Trolley 1 elastically collides with trolley 2 that was at rest on the track. The diagram describes the force (F) exerted on trolley 2 during the collision as a function of time. After the collision trolley 2 moves at a velocity of 1.25 m/s. Calculate the mass of trolley 2.



80. How many different words (with or without meaning) can be obtained by rearranging the letters of the word MATEMATIKA?