1. Assume that G is a graph consisting of 101 nodes and 100 edges. Which of the following could be true?
   1. G contains some cycles.
   It could be true. For example cycle of 100 nodes and 100 edges and one isolated node.
   2. G doesn't contain a cycle
   It could be true. For example the chain of 101 nodes and 100 edges.
   3. G is a tree
   It could be true. The same example as case 2.
   4. G is a disconnected graph
   It could be true. The same example as case 1.

2. ABC -> ABCBB -> AB -> BBAB -> BAABAB -> BAB -> empty string. It is not possible to use less operations, because it is necessary to apply at least three operations of type 1, and for deleting 9 letters, it is necessary to have at least three operations of type 2.
   Answer: 6.

3. Which of the following is true?

   1. The DNS used by OS allows a computer to identify other computers.
   Incorrect, because DNS (Domain name system) is not a part of OS (operating system).

   2. In order to connect to the internet, each computer needs a unique numerical code, which is known as IP address.
   Correct

   3. Internet Protocol version 4 (IPv4) defines an IP address as a 32-bit number, but IPv6 defines an IP address as a 64-bit number.
   Incorrect, because IPv6 defines an IP address as a 128-bit number.

4. Using 10 binary digits it's possible to represent numbers from 0 to $210 - 1 = 1023$.
   Answer: 1023.

5. This program takes three strings and concatenate them. The first string represents an input string without first character. The second string consists of two characters from the beginning of the input string. The third string is formed of every third character of the input starting from the beginning.
   Answer: egapolismemal.

6. Determine the output of the following program:

```
#include <bits/stdc++.h>
using namespace std;
const int n = 100;
int answer = 0;
void f(int l, int r, int ql, int qr) {
    ++answer;
    if (qr < l || ql > r)
        return;
    if (ql <= l && qr >= r)
        return;
    int mid = (l + r) >> 1;
    f(l, mid, ql, qr);
    f(mid + 1, r, ql, qr);
}
int main() {
    int l = 2, r = 33;
    f(1, n, l, r);
    cout << answer << endl;
}
```

   Answer: 25.

7. A tree of the height one contains one node. A tree of the height two contains at most 3 nodes, a tree of the height 3 contains at most 7 nodes, ..., a tree of the height k contains at most 2k – 1 nodes. $210 - 1 < 2019 < 211 - 1$, so the minimal possible height of the tree of 2019 nodes is equal to 11.
   Answer: 11.

8. If $n = 1$, then Andrew wins in one turn. If $n = 2$, then the first turn may be just 1, thus Bob wins. If $n = 3$, then the first turn may be just 1 and Bob loses after that, so Andrew wins. If $n = 4$, there are two turns and Andrew must choose number 2 in order to win. If $n = 5$, then the first turn may be just 1 and Andrew loses, so if n= 6 he wins and so on. Because all divisors of an odd number are odd, then subtraction of an odd number from odd number will get an even number. And if $n > 3$, then Andrew loses for all odd numbers n and wins for all even numbers n, because he can choose 1 to turn.
   Answer: Andrew wins for numbers 3, 2018, 2020.

9. There are at most 6 edges between 4 different vertices, so there are $2^6 = 64$ possible graphs there. Let's calculate the number of disconnected graphs:
   • 3 vertices are connected (by 4 different cases) plus one separated vertex;
   there are $4 \times 4 = 16$ graphs;
   • two pairs connected vertices – there are 3 graphs;
   • two vertices are connected and two vertices are not connected: there are 6 graphs;
   • there are no edges at all – introduce 1 graph.
   Answer: $64 - (16 + 3 + 6 + 1) = 38$.

10. Using two bits, it's possible to store at most 4 elements. The number of bits needed to represent the 3 elements are 2. For storing two elements, we need 1 bit. If we store 5 elements, then we need at least 3 bits.
    Answer: 1 (1, 2, 3).

11. Quick sort is a commonly used algorithm, its time complexity on average is $O(nlogn)$, time complexity for best possible input is $O(nlogn)$, time complexity for worst possible input is $O(n^2)$.
    Answer: 3.

    You can further explore relation between Moscow State University and quick sort https://cs.stanford.edu 09/tony-hoare/quicksort.html

12. Function what converts number from string type to integer type, so result for "180" is 180, but for "AB" the result is
    $(65 - 48) * 10 + 66 - 48 = 188$.
    Answer: 188.

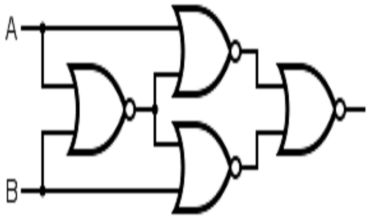
13. 1) $1101_2 + 1011\ 0001\ 1100_2 - 10\ 0110_2 = 1011\ 0000\ 0011_2 = B03_{16} = 2819_{10}$
    2) $1010\ 1011\ 1000_2 + 1\ 010\ 111_2 + 1000_2 = 101100010111_2 = B17_{16} = 2839_{10}$
    Answer: 2839.

14. The first and the second digits of the year are equal to a reverse day number. Maximal possible value before 2001 is 13. Another two digits are a month number in reverse order. Maximal possible value is 90 (from 09), but there is no 31st of September, so the answer is 1380.
    Answer: 1380.

15. In decimal, the sequence looks like 3, 9, 27, 81, i.e. 31, 32, 33, 34. Next number is 24310 = 3638.
    Answer: 363.

16. Note that $x \times y = gcd(x, y) \times lcm(x, y)$
Thus,
$x \times y = 15 \times 150 = 2250$
$x + y = 105$
$(105 - y) \times y = 2250$
$y2 - 105y + 2250 = 0$
$(y - 75)(y - 30) = 0$
There are two solutions:
$y = 75, x = 30$
$y = 30, x = 75$
$|x - y| = 45$
Answer: 45.

17. not (A xor B) = (A NOR (B NOR B)) NOR (A NOR B) NOR B) = (A NOR (A NOR B)) NOR (B NOR (A NOR B))
So, XNOR gate:



Answer: 4.

18. How many asterisks will be printed, if we call binom(7,4)?
Answer: 69.

19. This question is the equivalent to the problem of counting derangements, a permutation of the elements of a set, such that no element appears in its original position. We denote $f(n)$ as the amount of possible derangements. Suppose that there are n people who are numbered $1, 2, ..., n$. Let there be n gifts also numbered $1, 2, ..., n$. Assume that the first person takes gift i. There are $n - 1$ ways for the first person to make such a choice. There are now two possibilities, depending on whether or not person i takes gift 1 in return:
If person i does not take the gift 1, this case is equivalent to solving the problem with $n - 1$ persons and $n - 1$ gifts: each of the remaining n - 1 people has precisely 1 "forbidden" choice from among the remaining $n - 1$ gifts (i's forbidden choice is gift 1).
If person i takes the gift 1, then the problem reduces to $n - 2$ persons and $n - 2$ gifts.
Thus, we can write $f(n) = (n - 1)(f(n - 1) + f(n - 2))$
$f(1) = 0$
$f(2) = 1$
$f(3) = (3 - 1)(1 + 0) = 2$
$f(4) = (4 - 1)(2 + 1) = 9$
$f(5) = (5 - 1)(9 + 2) = 44$
$f(6) = (6 - 1)(44 + 9) = 265$
$f(7) = (7 - 1)(265 + 44) = 1854$
Answer: 1854.