Blitz. Short solutions (physics).

№1

Using the theorem of kinetic energy change

$$A_F + A_{тр} = \Delta E.$$

We have:

$$FS\cos\alpha - F_{тр}S = \Delta E.$$

Expressing the requested force and calculating we get $F_{fr} = 55$ N.

№2

The heat power is $P_1 = I^2R$, where I is calculated as $I = \frac{P}{U}$.

Performing calculations: $P_1 = \frac{P^2}{U^2}R = 2$ MW.

№ 3

Using energy conservation law:

$$mgh + \frac{mv_0^2}{2} = \frac{mgh}{2} + E_{loss}.$$

Expressing the requested energy loss and calculating we get $E_{loss} = 340$ J.

№4

Using known formula for period (frequency) of the simple gravity pendulum we get

$$\nu = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = 411 \text{ mHz}.$$

№5

The force components along the axis perpendicular to the string:

$$mg\sin\alpha = ma_{ц}\cos\alpha.$$

Using the formula for centripetal acceleration we write

$$a_c = 4\pi^2\nu^2 l\sin\alpha.$$

Substituting this equation into the previous formula and using that frequency is twice bigger we get: $\cos\beta = \frac{\cos\alpha}{4}$, откуда $\beta = 77,5^0$.

№6

The conservation of momentum $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{u}$.

Squaring both sides of the equation and using that vehicles velocities were perpendicular before the collision, i.e. $(\vec{v}_1 \cdot \vec{v}_2) = 0$, we get the final velocity:

$$u = \frac{\sqrt{(m_1v_1)^2 + (m_2v_2)^2}}{m_1 + m_2} \approx 12{,}87 \text{ m/s}.$$

Finally, we get the percentage of energy lost:

$$\alpha = \frac{\frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} - \frac{(m_1+m_2)u^2}{2}}{\frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2}} = \frac{m_1v_1^2 + m_2v_2^2 - (m_1+m_2)u^2}{m_1v_1^2 + m_2v_2^2}.$$

The numerical result is $\alpha = 49\%$.

№7

Let v be speed of the sprinter in the second part of the race. Then the average speed in the first part is $v_{avg} = \frac{v}{2} = \frac{l_1}{t_1}$; and $t_2 = \frac{l_2}{v}$. From the combined equation: $\frac{2l_1}{v} + \frac{l_2}{v} = t$ we get $v = 10$ m/s.

№ 8

For the acceleration of the body at an arbitrary point inside the tunnel we write:

$$a = G\frac{\frac{4}{3}\pi r^3\rho}{r^2} = \frac{4}{3}G\rho\pi r.$$

The force is proportional to the displacement from the equilibrium position and is restoring, that implies: $\ddot{x} = -\frac{4}{3}G\rho\pi x$, therefore, $\omega^2 = \frac{4}{3}\ G\rho\pi = \frac{g}{R}$.

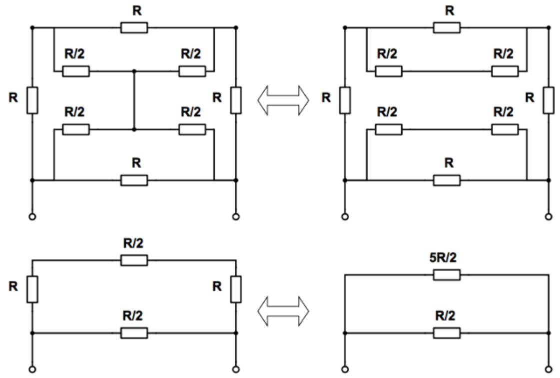
The travel time is half of oscillations period: $t = \frac{\pi}{\omega} = \pi\sqrt{\frac{R}{g}} \approx 41{,}9$ min.

№ 9

The cirquit can be redrawn as shown in the figures.

The resistance in question is:

$$R_{total} = \frac{5}{12}R = 5\ \Omega.$$



№ 10

According to the Snell law the refraction at the water surface is $\sin\alpha\ = n\sin\beta$, where α – incidence angle, β – refraction angle. $\sin\beta = \frac{2}{3}\sin\alpha$, therefore, $\sin\beta_{max} = \frac{2}{3}$ – when light enters the water almost tangentially. Shadow's radius is $R\ = r\ +\ H\text{tg}\beta$.

$$R_{max} = r\ + H\text{tg}\beta_{max} = r + \frac{2/3}{\sqrt{1 - 4/9}}H = r + \frac{2}{\sqrt{5}}H =\ 1{,}9\ \text{m}.$$

№ 11

Using the conservation of energy when the ball is: $\frac{k\Delta l_1^2}{2} = mg(l + \Delta l_1)$. We get the second-order equation and using the Hooke's law $mg =\ k\Delta l$, therefore

$$\Delta l_1 = \Delta l + \sqrt{\Delta l(\Delta l + 2l)} = 50\ \text{cm}.$$

Finally, $l_{max}\ = l + \Delta l_1 = 125$ cm.

№ 12

The expression for the maximum height is: $h\ = \frac{v_0^2\sin^2\alpha}{2g}$. Calculating for different angles: $\frac{h_1}{h_2} = 3$.

№ 13

In the first case the force is: $F\ = \frac{GM^2}{R^2}$, when the third ball is added the force defined by its presence is 4 times greater. Therefore, the resulting force is: $F_{res} =\ F\ +\ 4F = 5F$, i.e. the answer is 5.

№ 14

The process has two steps:

1. Due to the quick charge redistribution and voltage initialization some part of energy will be lost to the radiation (in the form of the spark).
2. The energy stored in capacitors at the same voltage will be released on the resistor.

The initial energy is $W = \frac{cu_0^2}{2}$. The charge is conserved, therefore: $cu_0 = cu + 3cu$, we find the voltage on the capacitors to be $u = u_0/4$. The energy of the system just after charge redistribution: $W_к = \frac{cu_0^2}{8} = \frac{W}{4}$. The final answer is 25%.

№ 15

Consider a small work done by the friction force dA_{fr} when the body is travelling along the small inclined part dS. Let the inclination angle be α. The friction force (as the Newton's second law implies) is $F_{fr} = \mu mg\cos\alpha$.
Then the work is:

$$dA_{fr} = -F_{fr}\cdot ds = -\mu mg\, ds\cos\alpha = -\mu mg\, dl\,.$$

Here dl – projection of ds on the horizontal axis.
Then the total work of friction force is $A_{fr} = -\mu mgl$; here l – horizontal component of the displacement.
From the conservation of energy, we get for each experiment

$$mgh = \mu mg(L + s_1),$$
$$2mgh = \mu mg(L + s_2),$$
$$3mgh = \mu mg(L + s_3).$$

Here L – projection of each inclined plane on the horizontal axis. Finally, $s_3 = 2s_2 - s_1 = 11$ m.

№ 16

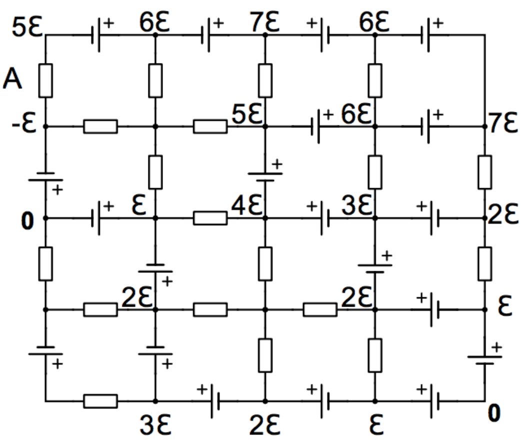
The conservation of energy gives $\frac{mv_0^2}{2} = mg\cdot 2R + \frac{mv^2}{2}$, where v – the speed at the top of the loop. At this point the centripetal acceleration is equal to the free fall acceleration $g = \frac{v^2}{R}$. Finally, $v_0^2 = 5gR$. $v_0 = 10$ m/s.

№ 17

When the mirror is rotated by a small angle Δ the beam is rotated by 2Δ. If the beam moves along the cylindrical screen of radius $R = PQ$, the speed of the spot is $v_p = 2\omega R$. But the beam moves on the plane screen, therefore the speed of the spot P is $v = v_p/\cos 60^0$. Finally,

$$\omega = \frac{v}{4R} = 1{,}5\ \frac{\text{rad}}{\text{s}}.$$

№ 18



Let's note the potentials of the points as show in the figure (to do this we move along the circuit from the bottom right point passing through the batteries).
The voltage of the resistor *A* is 5*E*. Therefore, the current is

$$I = \frac{6E}{R} = 6\text{ A}.$$

№ 19

The rings will be attracting in two options.
The first is when the magnetic field strength $\vec{B}$, created by the first ring at the center of the second loop is directed towards the first ring AND the induced current is directed so that the

magnetic field created by it is directed as external magnetic field is. This case corresponds to $I > 0$, $\frac{dI}{dt} < 0$.
The second option is inverse. If both the external magnetic field and induced current change the direction, the Ampere's force acting on the second ring keeps its direction. This case corresponds to $I < 0$, $\frac{dI}{dt} > 0$.
Therefore, the answer is – time periods 2 and 4.
Note: At the center of the second ring the magnetic field is perpendicular to its plane, but there is a component of the field on the ring's "edge" that is within the plane of the ring. That component leads to the appearance of the attraction and repulsion.

№ 20
The change of momentum of the second trolley is

$$\Delta p = \int F dt = m_2 \Delta v.$$

The value $\int F dt = 5$ N·s can be found as the area under the graph. Since before collision the second trolley was at rest, $\Delta v = v_2$, therefore we get $m_2 = \frac{\int F dt}{v_2} = 4$ kg.