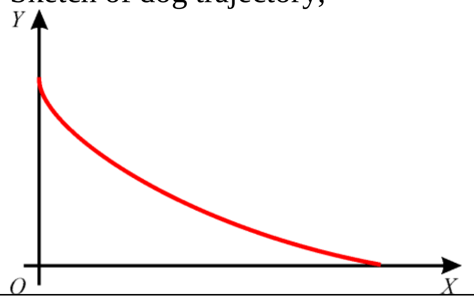
Problem 1

City ____________________ **Name** ____

№	ANSWER	Maximum Score	Score
1	Acceleration points toward the owner in the negative direction of OY, $a_0^{\text{dog}} = V_0^2 / l$.	**1**	
2	Magnitude of dog's velocity at A, $V_A^{\text{dog}} = V_0 \cos\alpha = \dfrac{V_0}{\sqrt{2}}$.	**1**	
	Dog acceleration at A is directed along OX, $\vec{a}_A^{\text{dog}} = \dfrac{V_0^2}{l\sqrt{2}} \cdot \vec{e}_x$.	**2**	
3	Dog trajectory in parametric form, $x(\alpha) = l \cdot \left\{ \ln\left[\cot\left(\dfrac{\alpha}{2} \right) \right] - \cos\alpha \right\}$ and $y(\alpha) = l \cdot \sin\alpha$; in explicit form: $x(y) = l \cdot \ln\left(\dfrac{l + \sqrt{l^2 - y^2}}{y} \right) - \sqrt{l^2 - y^2}$.	**5 (for parametric form: 3 for *x* + 2 for *y*)**	
	Sketch of dog trajectory, 	**1**	
4	Dog law of motion, $x(t) = V_0 t - l \cdot \tanh\left(\dfrac{V_0 t}{l} \right)$ and $y(t) = l \cdot \cosh^{-1}\left(\dfrac{V_0 t}{l} \right)$.	**5 = 3 for *x* and 2 for *y***	
	Time dependence of dog speed, $V^{\text{dog}}(t) = V_0 \cdot \tanh\left(\dfrac{V_0 t}{l} \right)$.	**3**	
5	Horizontal component of force exerted on dog by the ground versus time, $F(t) = \dfrac{mV_0^2}{l} \cdot \cosh^{-1}\left(\dfrac{V_0 t}{l} \right)$.	**4**	
6	Dog law of motion after a long time (for $t >> \dfrac{l}{V_0}$), $x(t) \approx V_0 t - l$, $y(t) \approx 0$.	**2**	
7	Minimum distance between the dog and the fly after a long time, $r_{\min} = l\left[1 - \ln\left(1 + \left(e - \sqrt{e^2 - 1} \right)^2 \right) \right] \approx 0{,}964 l$.	**4 (if mistake is only in integral calculation - 2)**	
8	Magnitude of fly's acceleration at A, $a_A^{\text{fly}} = \dfrac{V_0^2}{l}$.	**2**	
	TOTAL	**30**	

Problem 2

City ____________________ **Name** ____

№	ANSWER	Maximum Score	Score
1.1.	$\begin{cases} I_1 = (\mathcal{E} - U)/R \\ \alpha I_2^2 = U + \mathcal{E} \\ 2\alpha I_3^2 = U \\ I_4 = I_0 f(1-x) \\ I_2 + I_3 = I_1 + I_4 \end{cases}$, where $x \equiv \frac{U}{\mathcal{E}}$ and $f()$ is the diode I-V function. Any equivalent set of equations is accepted.	**1**	
1.2.	$I_\text{A} = (0,267 \pm 0,005)$ A ; less accurate answer: $I_\text{A} = (0,27 \pm 0,02)$ A ; «crude» answer: from 0,2 A to 0,3 A.	**4 (3) (1)**	
2.1.	$t_0 = \sqrt{\alpha C}\int_0^{q_0} \frac{dq}{\sqrt{q}} = 2\sqrt{\alpha C q_0}$	**2**	
2.2.	Equation written: $\tau = rC$	**1**	
2.3.	$t = rC \ln\left(\frac{I_0}{I}\right) + 2\alpha C(I_0 - I)$	**3**	
2.4.	$t = \tau \ln\left(\frac{\sqrt{(t_0/\tau)^2+1}-1}{\sqrt{(t_0/\tau\sqrt{n})^2+1}-1}\right) + \sqrt{t_0^2+\tau^2} - \sqrt{\frac{t_0^2}{n}+\tau^2}$ or $t \approx t_0 + \tau \ln\left(\frac{2n\tau}{e t_0}\right)$. Numerical answer: 25-26 ms (24-28 ms).	**4 = 2 for equation (in any form) + 2 (1) for numerical answer**	
3.1.	$I(q) = \sqrt{\frac{1}{\alpha C}\left\{\frac{L}{2\alpha} + q - \left(\frac{L}{2\alpha} + q_0\right)\cdot \exp\left(\frac{2\alpha}{L}(q-q_0)\right)\right\}}$	**4**	
3.2.	$\frac{Q_1}{E_0} \approx 10\%$, acceptable interval from 9,5% to 10,5% (from 9% to 11%)	**4 (2)**	
3.3.	$\frac{Q_1'}{E_0} = 19\%$	**3**	
3.4.	$\frac{Q_2}{E_0} \approx 27\%$, acceptable interval from 26% to 27,5% (from 25% to 29%)	**4 (2)**	
	TOTAL	**30**	

Problem 3

City ____________________ **Name** ____

№	ANSWER	Maximum Score	Score
1.1.	Density of heat outflow from the planet-ocean depths, $q_0 = \sigma T_1^4 \approx 0{,}35\ \mathrm{W/m^2}$; Allowed range (0,34 ÷ 0,36) $\mathrm{W/m^2}$.	**3 = (2 for equation) + (1 for num. value)**	
1.2.	Ice sheet thickness at pole, $H_1 = \frac{A}{\sigma T_1^4}(T_{in} - T_1)\left(1 - \frac{\beta}{2}(T_{in} + T_1)\right) \approx 2250\ \mathrm{m}$, here $A = 5{,}40\ \mathrm{W/(m \cdot K)}$, $\beta = (1/465)\ \mathrm{K^{-1}}$. Allowed range (2200 ÷ 2300) m.	**3 = (2 for equation) + (1 for num. value)**	
1.3.	Ice sheet thickness on equator in area of maximum temperature $H_2 = \frac{A}{\sigma T_1^4}(T_{in} - T_2)\left(1 - \frac{\beta}{2}(T_{in} + T_2)\right) \approx 1600\ \mathrm{m}$, here $A = 5{,}40\ \mathrm{W/(m \cdot K)}$, $\beta = (1/465)\ \mathrm{K^{-1}}$. Allowed range (1550 ÷ 1650) m.	**2 = (1 for equation) + (1 for num. value)**	
1.4.	Temperature of star photosphere $T_S = \sqrt[4]{\frac{r_0^2}{R_X^2}(T_2^4 - T_1^4)} \approx 3100\ \mathrm{K}$. Allowed range (3000 ÷ 3200) K.	**3 = (2 for equation) + (1 for num value)**	
1.5.	Maximum daytime temperature on the planet surface versus latitude: $T(\theta) = \sqrt[4]{T_1^4 + (T_2^4 - T_1^4)\cos\theta}$	**2**	
1.6.	$\Lambda_X \approx 929$ nm. Allowed range (928 ÷ 930).	**1**	
2.1.	Time of water rising in polynya to the new equilibrium level, $\tau \approx \sqrt{\frac{2\rho H_2}{\rho_0 g}} \approx 54\ \mathrm{s}$, where $H_2 = \frac{A}{\sigma T_1^4}(T_{in} - T_2)\left(1 - \frac{\beta}{2}(T_{in} + T_2)\right) \approx 1600\ \mathrm{m}$. Allowed range (50 ÷ 60) (40 ÷ 70)*	**3 = (1 for equation) + (2 (1) for num. value)**	
2.2.	Ice crust thickness in polynya immediately after the freezing of surface layer, $h_0 = \frac{Lp_3}{\rho\lambda g} \approx 4{,}5\ m$, where p_3 = 610 Pa is the pressure at triple point of water. Allowed range (4,3 ÷ 4,6) (4 ÷ 5)*	**5 = (3 for equation) + (2 (1) for num. value)**	
2.3.	Crater depth, $h_c \approx 160$ m. Allowed range (150 ÷ 170)	**2**	
3.1.	Ice layer thickness will increase two-fold compared to the initial thickness h_0 in $t_1 \approx \frac{3\lambda\rho}{2q_0 H_2}h_0^2 \approx 1{,}66 \cdot 10^7\ \mathrm{s} \approx$ ≈ 192 **Earth** days. Allowed range (190 ÷ 195) (180 ÷ 200)*	**5 = (3 for equation) + (2 (1) for num. value)**	

3.2.	Age of polynya in which ice crust is h = 100 m thick, $t_2 \approx \frac{\lambda\rho}{q_0 H_2}\left(\frac{h^2}{2} - \frac{h_0^2}{2}\right) \approx 2{,}7 \cdot 10^9$ s $\approx$ $\approx$ 87 **Earth** years. Allowed range (85 ÷ 95) (80 ÷ 100)*	**4 = (2 for equation) + (2 (1) for num. value)**	
3.3.	Crude estimate for the age of polynya in which the bottom of ice crust reached that of the surrounding ice, $t_3 \approx \frac{\lambda\rho H_2}{q_0} \ln\left(\frac{H_2}{H_2 - H_0}\right) \approx 3{,}2 \cdot 10^{12}$ s $\approx$ $\approx$ 102 thousand **Earth** years. Allowed range (100 ÷ 105) (95 ÷ 110) Accurate answer, $t_3 = \frac{\lambda\rho}{q_0}\left[H_2 \cdot \ln\left(\frac{H_2 - h_0}{H_2 - H(t)}\right) - H(t) + h_0\right] \approx$ $\approx$ 62 thousand **Earth** years. Allowed range (60 ÷ 65).	**7 = 4 for equation + 3 for num. value (100 ÷ 105)** **5 = (3 for equation) + (2 for num. value 95 ÷ 110),** **the accurate answer also scores as 7 = 4 + 3**	
TOTAL		**40**	

*The second range corresponds to the score in parenthesis.