# Experiment. Introduction. Part 1.

## Electromagnetic Waves

Varying electric field in space generates varying magnetic field. Varying magnetic field, in turn, generates varying electric field. Combination of these phenomena leads to existence of electromagnetic wave. Electromagnetic wave is a propagating in space and varying in time electromagnetic oscillation, i.e. oscillating electric field ($\vec{E}$) and magnetic induction ($\vec{B}$). Electromagnetic wave is transversal for both vectors, i.e. directions of oscillation of $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of wave propagation and the directions of oscillation of $\vec{E}$ and $\vec{B}$ are orthogonal (see Fig. 1). For simplicity, only the vector $\vec{E}$ is usually discussed by implicitly assuming that wave propagation itself is caused by interconnection of varying magnetic and electric fields.

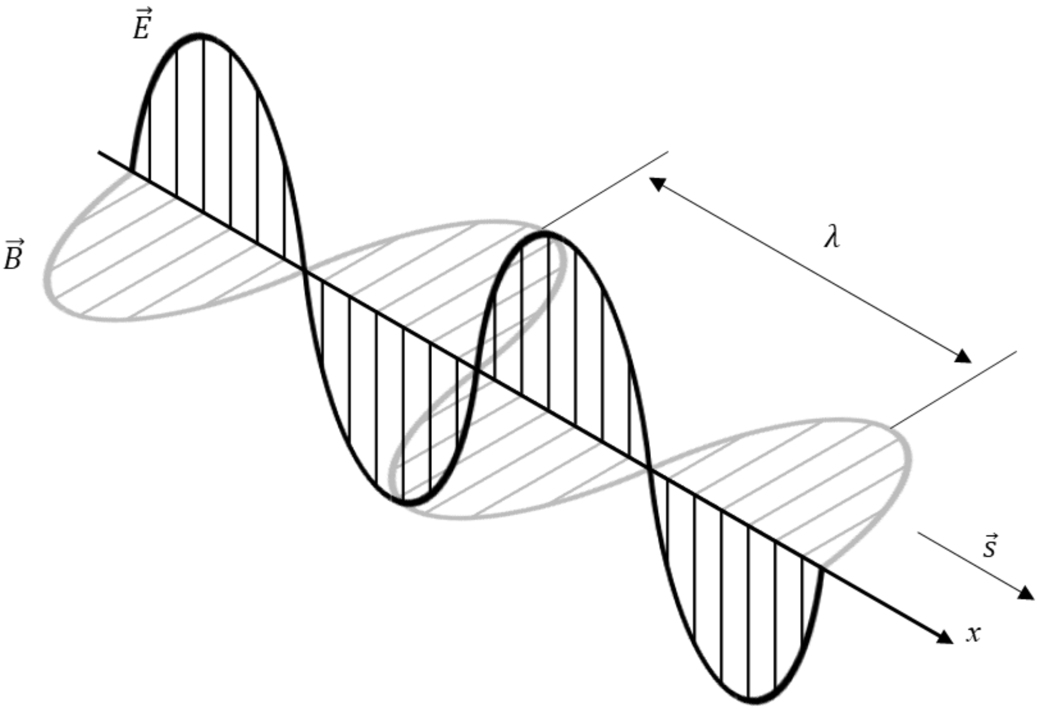


*Fig. 1. Electromagnetic wave.*
*$\vec{E}$ is electric field vector, $\vec{B}$ is vector of magnetic induction,*
*$\vec{s}$ is direction of propagation of electromagnetic wave, and $\lambda$ is wavelength.*

If oscillation of electric field at each point of space can be described by a harmonic function, the wave is called monochromatic. If electric field vectors are the same everywhere in a plane perpendicular to the direction of wave propagation, it is plane wave. In a plane wave the electric field oscillates with a period $T$ at any point of space and the oscillation phase varies linearly when moving in the direction of $\vec{s}$ from a point with a coordinate $x$ to a point with another coordinate.

Let us project an electric field vector on the direction of oscillation of this vector at every point. Then for the component of the electric field vector of a wave propagating along the axis $x$, one can write:

$$E(x,t) = E_0 cos\left(\frac{2\pi t}{T} - kx + \varphi_0\right), \tag{1}$$

where $t$ is time, $T$ is a period, and $\varphi_0$ is an initial phase of oscillation (at $t = 0$ at the points with coordinate $x = 0$). The quantity $k$ is known as wavenumber, it defines the phase lag between one point on the axis $x$ and another point on this axis. The dependence of $E(x)$ is a sinusoid. A period of the sinusoid is called the wavelength and is denoted by the Greek letter λ. The oscillation phase varies by $2\pi$ under translation along $x$ by the wavelength. Thus, the wavenumber and the wavelength are related as

$$2\pi = k\lambda. \tag{2}$$

Therefore, the wavenumber is

$$k = \frac{2\pi}{\lambda}. \tag{3}$$

For notational convenience one can introduce the angular frequency of electric field oscillations:

$$\omega = \frac{2\pi}{T}. \tag{4}$$

The plot of the harmonic function $E(x,t)$ being considered varies with time. One can imagine this variation as «translation» of the sinusoid along the axis $x$. The translation proceeds at some velocity $v$. In other words, an oscillation with a certain phase is observed at the point with coordinate $x$ at a time $t$ and after a time interval $\Delta t$ it is observed at the point with coordinate $x + v\Delta t$. Thus, the velocity $v$ specifies displacement of points with the same oscillation phase along the direction of wave propagation. By this reason, the velocity $v$ is called phase velocity. The coordinate of a point with a certain oscillation phase changes by the single wavelength per period. Hence, the phase velocity is:

$$v = \frac{\lambda}{T}, \tag{5}$$

or in terms of the wavenumber and angular frequency:

$$v = \frac{\omega}{k}. \tag{6}$$

In this problem we will deal with electromagnetic waves in the optical range with wavelengths from 2 μm to 10 nm. Often these electromagnetic waves are referred to as light.

A plane monochromatic wave represents the simplest example of a wave process. For instance, the mathematical expression of electric field of a spherical wave propagating in all directions from a point-like source is:

$$E = \frac{E_0}{r} cos(\omega t - kr + \varphi_0), \quad (7)$$

where $r$ is the length of a radius vector drawn from the point-like source to a point of observation.

In general case there are several waves propagating in space in various directions from different sources with various frequencies. At any given moment, the vector of electric field at a point of space can be found by using the principle of superposition. That is, electric fields of the waves coming from different sources will add up as vectors at every point.

For a given direction of wave propagation the electric field vector can oscillate along different directions perpendicular to the vector $\vec{s}$. A light produced by a typical source is seen in a certain direction of propagation as a set of waves with various directions of oscillation of electric field and various initial oscillation phases unrelated to each other. Tracing the electric field of such a wave at some point one would see that the electric field vector varies chaotically in all possible directions in the plane perpendicular to the direction of wave propagation. Such a light is called naturally polarized. If the electric field vector oscillates strictly along the single direction, this light is called linearly polarized. The plane in which both the vector of light propagation and the direction of oscillation $\vec{E}$ lie is called the polarization plane. Linearly polarized light can be obtained from a natural light by transmitting it through a linear polarizer (LP). This is an optical device transmitting only the component of incident light which is polarized in a certain plane (i.e. a linear polarizer is made of a material letting a certain component of vector $\vec{E}$ to pass and absorbing the other one). This plane is called the transmission plane (TP) of a linear polarizer. The second component which is perpendicular to TP is absorbed by the polarizer. The operation principle of LP is illustrated in Fig. 2 (the wave propagates from left to right).

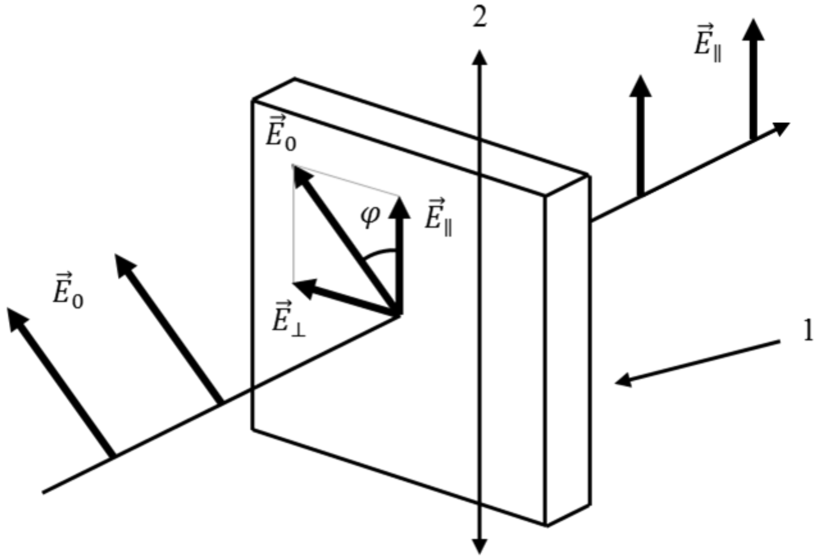


*Fig. 2. Passing of light through linear polarizer.*
*1 is linear polarizer and 2 is polarization plane of linear polarizer.*

Here $\vec{E}_0$ is the amplitude of electric field of incident wave at a certain moment of time, $\vec{E}_{\parallel}$ is the amplitude of the wave transmitted through LP, $|\vec{E}_{\parallel}| = |\vec{E}_0| \cdot \cos\varphi$, and φ is the angle between $\vec{E}_0$ and $\vec{E}_{\parallel}$.

**According to the theory of electromagnetic waves the light intensity is proportional to the time average of the square of electric field.** This allows one to determine the relation between the intensities of the transmitted and incident waves and the angle φ (Malus's law):

$$I_{\text{transmitted}} = I_{\text{incident}} \cdot \cos^2\varphi. \quad (8)$$

A plane monochromatic wave which is infinite in space and in time does not exist in reality. This is a model that helps in physical reasoning and mathematical description. A wave has always the source which started generating the wave at a certain moment of time. Therefore, the wave temporal extent is not infinite. On the other hand, electromagnetic waves propagate at a finite although a very high speed. So, the spacial extent of a wave is also limited. Of course, this is not the only reason for a wave to be bounded, the waves can be absorbed or reflected by some objects or inhomogeneities. Let us take a closer look at the process of wave propagation.

The «direction of propagation of phase» is described by vector $\vec{k}$ which magnitude is the wavenumber and the direction is perpendicular to the surface of constant phase at a given point. Such a surface is called a phase surface or a wave surface. For instance, a plane is the wave surface for a plane wave. The wave surface at a given moment of time is called a wavefront. That is, wave surfaces were the wavefronts at the previous moments of time. The concept of wave surface allows one to explain quite easily the phenomenon of light propagation in a medium. This idea was first proposed by C.~Huygens and later it was further developed by A.~J.~Fresnel. Let a wave to have a wavefront depicted in Fig. 3 by the line 1. Suppose that every point of the wavefront is a source of secondary spherical waves. After a while the waves from the secondary sources will have moved forward and each wave forms its own wavefront. The envelope of these wavefronts (the line 2 in Fig. 3) is a new wavefront of the initial wave. Indeed, the phases of field oscillation coincide at the envelope of wavefronts of the secondary waves propagating from the secondary sources. Therefore, the sum of such in-phase oscillations will give the greater value of the field. At the points not at the envelope the oscillation phases of secondary waves are different and the resulting amplitude will be much less than that at the envelope. This principle also explains the absence of a backward wave from the secondary sources. The secondary waves propagating «forward» enter an unperturbed space and add up only among themselves. The secondary waves going «backward» enter the space where the initial wave is already present. In so doing, the secondary waves and the «original» wave mutually quench because they have opposite phases, so the propagating wave leaves behind the unperturbed space.

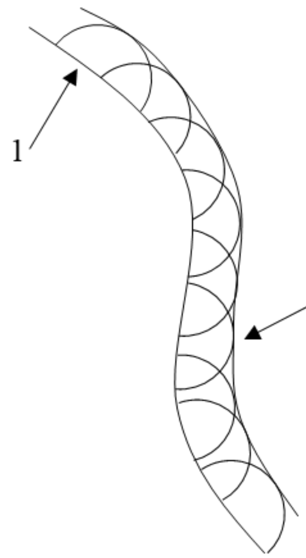


*Fig. 3. Huygens method of constructing wavefront by using secondary sources.*

A physical model of the secondary source in vacuum can be provided by the electric field itself which, by varying at a given point of space, generates a magnetic field which, in turn, generates electric field in the neighboring points. This mechanism of wave propagation in vacuum is characterized by the velocity $c = 3 \cdot 10^8$ m/s. This velocity is known as the velocity of light in vacuum.

A physical model of the secondary source in a medium must be supplemented with a mechanism of reemission by atoms and molecules. Atoms and molecules are polarized by the electric field, i.e. they acquire a dipole moment in the electric field direction. If the electric field varies with time, the dipole momenta of atoms and molecules vary as well and emit the secondary waves. There are plenty of polarization mechanisms of atoms and molecules and each of them is described by its own model. However, here we will only focus on the model of atomic polarization which can explain the phenomena of interest to us. Imagine an atom surrounded by an electron cloud (in a certain sense, this is the case from the quantum-mechanical viewpoint). The electron cloud is being deformed by the external electric field. In the simplest case of a small electric field and some symmetry of the atom surroundings, the induced dipole moment of the atom will be along the electric field and the amplitude of dipole moment oscillations will be proportional to the amplitude of electric field. Then the electric field of the reemitted wave will be parallel to the field of the original wave. However, the phase of the reemitted wave will lag behind the original wave phase since, unlike in the model of the secondary wave in vacuum, the electron cloud of atom possesses some inertia, so the dipole oscillations lag relative to the oscillations of the incident electric field. This would result in a phase lag of the reemitted wave relative to the original one. Such a lag affects the velocity of wave propagation in a medium and can be described by a refractive index $n$. Thus, the phase velocity $v$ of a wave in a medium equals the ratio of the velocity of light in vacuum to the refractive index of the medium:

$$v = \frac{c}{n}. \tag{9}$$

It is worth mentioning that if the electron cloud is considered as a harmonic oscillator with a certain eigenfrequency, its lag relative to the external force generated by external electric field will increase as the field frequency increases. This allows one to say that the velocity of wave propagation in a medium will decrease as its frequency grows. In other words, the refractive index will increase with the frequency. This is indeed the case for any material in the optical range (excluding the frequencies coinciding with the eigenfrequencies of oscillation of atomic/molecular electron clouds, i.e. in a region of absorption of electromagnetic waves by a material).

Two phenomena occur under electromagnetic wave incidence on an interface of different media - reflection and refraction. That is, the secondary waves generate a reflected wave propagating in the first medium and the refracted wave propagating in the second medium. It is natural that these waves have the same frequency as the incident wave. Phases of oscillation of electric field at any point at the interface must coincide for the incident and refracted waves, the field oscillations belong both to the incident and to refracted waves. If the incident wave is a plane wave, the refracted wave will also be a plane wave. Let us determine the phase shift between two sources $A$ and $B$ at the medium interface (see Fig. 4) which is due to the phase lag of the incident wave.

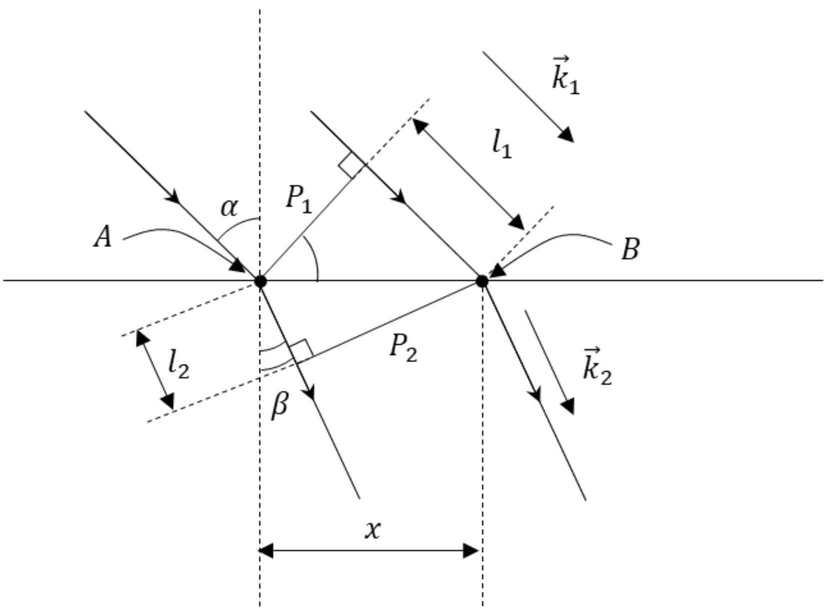


*Fig. 4. To derivation of law of refraction in isotropic medium.*

The plane $P_1$ is perpendicular to the vector $\vec{k}_1$ and is the plane of equal phases. Then for oscillations at the point $B$ there is a phase lag relative to the point $A$ by a product of the absolute value of the wave vector of incident wave and the distance $l_1$ (this is the displacement of the phase surface from the point $A$ to the point $B$):

$$\Delta\varphi = k_1 l_1. \tag{10}$$

The distance $l_1$ can be expressed via the distance between the sources $A$ and $B$ in terms of the angle of incidence. This gives for the phase shift:

$$\Delta\varphi = k_1 x \sin\alpha. \tag{11}$$

On the other hand, the same phase shift for the reflected wave can be expressed via the wavenumber of the reflected wave and the refractive index:

$$\Delta\varphi = k_2 x \sin\beta. \tag{12}$$

Thus, equating the phase shifts one determines the relation between the angles of incident and refracted waves:

$$k_1 \sin\alpha = k_2 \sin\beta. \tag{13}$$

Using the formula for the wavenumber one obtains the famous law of refraction:

$$\begin{gathered} \frac{\omega}{v_1}\sin\alpha = \frac{\omega}{v_2}\sin\beta, \\ n_1 \sin\alpha = n_2 \sin\beta. \end{gathered} \tag{14}$$

The formula (13) can be interpreted as the law of conservation of the wavevector component on the interface of two media.

## Propagation of light in anisotropic media.

Deformation of electron cloud in atom depends both on the external field and on the internal electric field of neighboring atoms. In some materials the process of displacement of electron cloud (the vector of induced dipole moment and its phase shift) depends both on the electric field magnitude and direction. In such media the velocity of wave propagation depends on direction. The propagation velocity will also depend on a wave polarization. These media are called anisotropic, their properties depend on a direction chosen in medium.

Among a plentiful of anisotropic materials there are the so-called uniaxial crystals. Such a crystal has a single specific direction in which light propagates as in a regular isotropic medium. This direction in an anisotropic crystal is called the optical axis. The optical axis coincides with a symmetry axis of the crystal. Further the optical axis on a diagram will be denoted by $z$.

If an electric field applied to such a crystal is parallel to the optical axis, the induced dipole moment of the atoms will be proportional to the electric field with some factor. If the electric field in this crystal is perpendicular to the optical axis, the induced dipole moment of atoms will be proportional to the electric field with another factor. The proportionality factor is the same for all directions of electric field in a plane perpendicular to the optical axis. In general case, when the electric field is at some angle to the axis $z$, the induced dipole moment of atoms can be calculated as a superposition of the response to two components of the field, one of which is parallel to $z$, and the other one is in the perpendicular plane. Notice that in general case the induced dipole moment is not parallel to the electric field. Similar reasoning can be given for the lag of oscillations of the induced dipole moment, this lag depends on the direction of alternating electric field. This results in a dependence of the velocity of electromagnetic waves in anisotropic materials on the direction of propagation.

When analyzing propagation of an electromagnetic wave in these materials, the wave is decomposed in two waves of different polarization called «ordinary» and «extraordinary». Let us call the plane formed by a wave vector and the optical axis as the principal optical plane.

The «ordinary» wave is a wave (subscript «o») which polarization plane is perpendicular to the principal optical axis of a crystal. For any ordinary wave the electric field vector $\vec{E}_o$ lies in a plane perpendicular to the optical axis (see Fig. 5(a)). Regardless of the direction of electric field in the plane, the response of the medium is the same, so the velocity of an ordinary wave in the material will be independent of the direction and the same will be true for the refractive index. It is from this fact the wave derives the name «ordinary», since for this wave polarization a uniaxial crystal behaves as an isotropic medium.

The «extraordinary» wave (subscript «e») is a wave which polarization plane coincides with the principal optical plane of the uniaxial crystal. The electric field vector of this wave $\vec{E}_e$ does not lie in a plane perpendicular to the crystal optical axis (see Fig. 5(b)). Therefore, the medium response depends on the direction of the wave propagation so that the wave velocity depends on the direction of propagation. Therefore, each direction in the crystal has its own refractive index. There is no difference between «ordinary» and «extraordinary» waves when light propagates along the crystal optical axis. In this case, the electric field of any wave lies in a plane perpendicular to the optical axis.

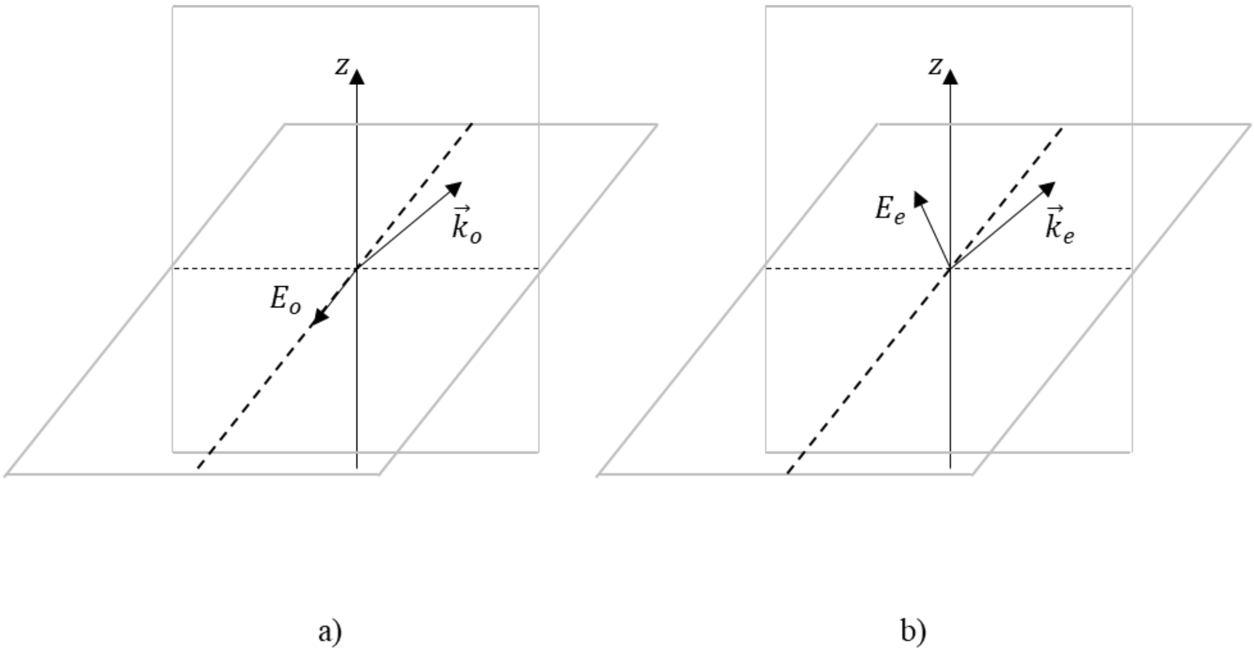


*Fig. 5. a) Direction of electric field vector of ordinary wave;*
*b) Direction of electric field vector of extraordinary wave.*

Thus, a naturally polarized wave incident on the surface of uniaxial crystal generates in the crystal two waves of different polarization (ordinary and extraordinary). The refractive indices for these waves are different, so they are refracted differently, which results in two directions of light propagation in the crystal. This phenomenon is called birefringence. It was discovered as a property of calcite (calcium carbonate $CaCO_3$). This mineral occurs in nature and possesses a strong birefringence effect. Birefringence can be also observed in a crystal of quartz although the difference in the refractive indices of «*o*» and «*e*» waves is not so large. Because of that it is almost impossible to observe the birefringence phenomenon in quartz with the naked eye. The birefringent crystals nowadays are grown in the labs, some of them do not occur naturally.

The refractive index of ordinary wave in the crystal is the same for all directions of propagation and is denoted by $n_o$. The refractive index for an extraordinary wave depends on the direction of propagation as follows. Let us draw a surface formed by the points separated from some center at a distance equal to the refractive index $n$ corresponding to this direction of propagation from the center. This surface is an ellipsoid called the refractive index ellipsoid. A similar surface for «ordinary» wave is the sphere. By dissecting this ellipsoid with a plane passing through the optical axis one obtains an ellipse (see Fig. 6). A similar shape for ordinary wave is a circle of radius $n_o$. Both shapes have the same size in the direction of optical axis, i.e. the circle radius equals to a semi-axis of the ellipse. The obtained ellipse is described by the following equation:

$$\frac{sin^2(\theta)}{n_o^2} + \frac{cos^2(\theta)}{n_e^2} = \frac{1}{n^2}, \quad (15)$$

where $\theta$ is the angle between the chosen direction and the optical axis and $n_e$ is the refractive index of extraordinary wave propagating perpendicular to the optical axis. The difference $n_e - n_o$ specifies birefringence of the crystal. The larger the difference, the greater is the birefringence effect. Crystals with $n_e - n_o > 0$ are called positive, and crystals with $n_e - n_o < 0$ negative. The calcite crystal is negative since it has $n_o = 1{,}6585$ and $n_e = 1{,}4863$. It is because of this big difference of the refractive indices the birefringence was discovered in calcite.

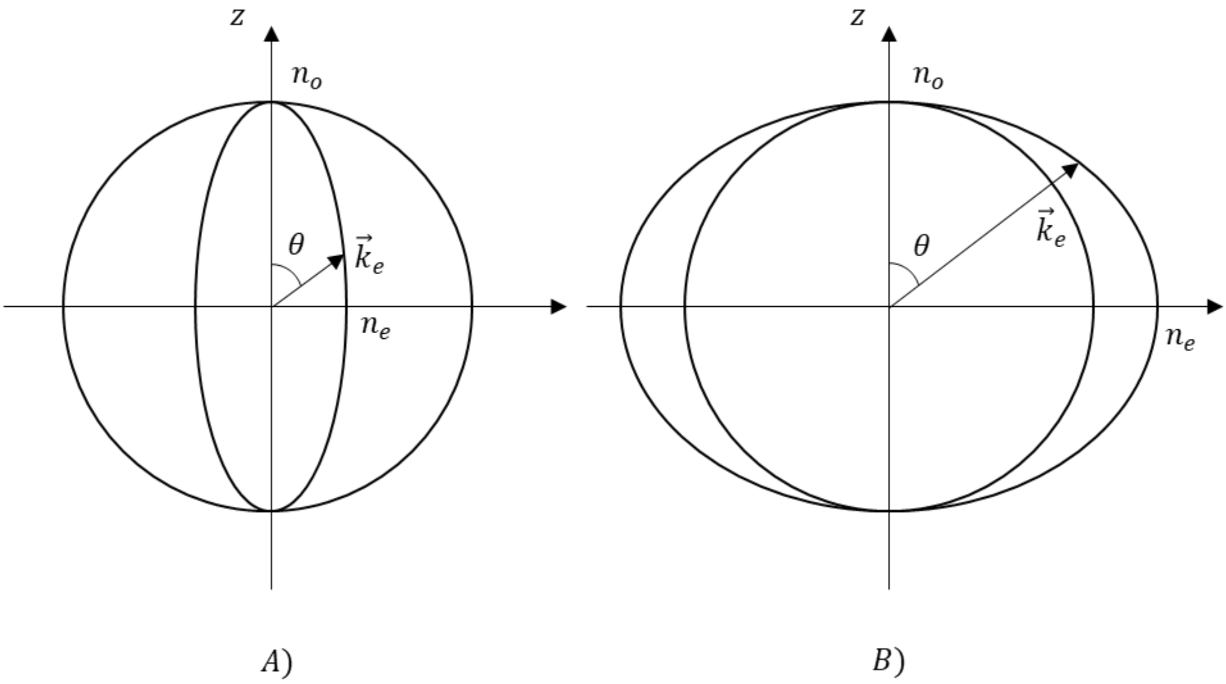


*Fig. 6. Refractive index ellipsoid cross section*
*A) Negative crystal; B) Positive crystal.*

In a sense, refraction in a birefringent crystal obeys Snell's law although there are some peculiarities. An ordinary wave is refracted as in an isotropic material. For an extraordinary wave the refractive index in Snell' law must be understood as a function of refraction angle. A graphical method provides the most illustrative representation of refraction at the interface between two media (see Fig. 7). Suppose a naturally polarized plane wave with a wavevector $\vec{k}_{inc}$ is incident on a birefringent crystal from an isotropic medium with a refractive index $n_{inc}$. The magnitude of the wavevector is $k = \frac{\omega}{c}$. The reflected wave will also propagate in the original medium and the magnitude of its wavevector is the same as the magnitude of the wavevector of the incident wave. Two waves will originate in the crystal: the ordinary, with the wavevector $\vec{k}_o$ , and extraordinary, with the wavevector $\vec{k}_e$. The magnitudes of these wavevectors are: $k_o = n_o \frac{\omega}{c}$ and $k_e = n \frac{\omega}{c}$. The components of these two vectors on a direction along the crystal surface coincide, see the above discussion of the isotropic case. The vector components can be calculated via the sines of the incidence (reflection) angle $\alpha$ and the refraction angles of ordinary $\beta_o$ and extraordinary $\beta_e$ waves. Thus, Snell's law can be written as:

$$n_{inc} sin\alpha = n_o sin\beta_o = n sin\beta_e, \tag{16}$$

where $n$ is the refractive index of extraordinary wave in the direction of its propagation with respect to the crystal optical axis.

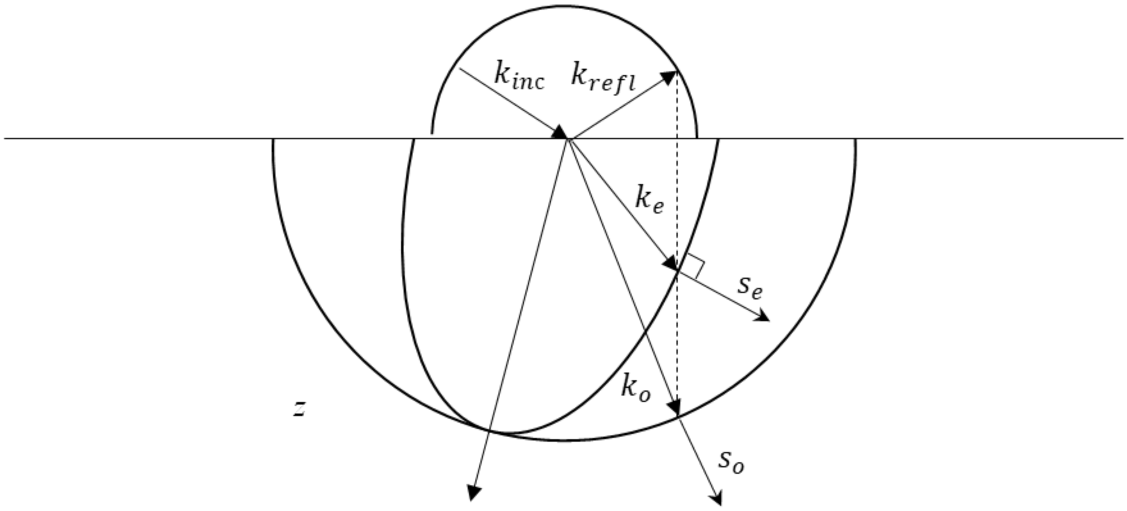


*Fig. 7. Finding out directions of propagation of refracted rays in uniaxial crystal.*

However, this does not exhaust the difference between refraction of light at the boundary of uniaxial crystal compared to refraction at the boundary of optically isotropic material. It turns out that in general case the direction of propagation $\vec{s}$ of extraordinary ray does not coincide with the direction of wavevector $\vec{k}$. The direction of ray propagation is understood as the direction of energy transfer by light. For instance, if the crystal had small scattering particles, the light scattered by them would leave a fluorescent trace which would allow one to follow the direction of energy transfer in the crystal. Obviously, it is this direction which determines the point of exit of the ray from the crystal. It is possible to prove that the direction of energy transfer by extraordinary ray, $\vec{s}_e$, is perpendicular to the surface of the refractive index ellipsoid of extraordinary ray at the point corresponding to the direction of $\vec{k}_e$. In general case, the directions of $\vec{s}_e$ and $\vec{k}_e$ are not the same. It is said that the rays of extraordinary wave are «walked-off».

Note that wave vectors of both refracted waves always lie in the incidence plane formed by the wavevector of incident wave and a normal to the interface between the media. However, the direction of propagation of extraordinary ray (vector $\vec{s}_e$) can be out of the plane. Indeed, if the crystal axis does not lie in the incidence plane, the perpendicular to the refractive index ellipsoid at the point of intersection with the direction of vector $\vec{k}_e$ will not lie in the incidence plane. Then the extraordinary ray will be walked-off not in the incidence plane.

In geometrical optics the direction of $\vec{s}_e$ should be regarded as the direction of propagation of the extraordinary ray while $\vec{k}_e$ should be used in calculation of the refraction angle $\beta_e$ and a walk-off of the ray from this direction. There is no walk-off of the ray corresponding to ordinary wave and the ray direction $\vec{s}_o$ coincides with the direction of $\vec{k}_o$. The extraordinary ray is not always walked-off. If vector $\vec{k}_e$ is parallel or perpendicular to the principal optical axis, there is no walk-off. In these cases, the angle between a perpendicular to the ellipsoid surface and the directions to these points from the ellipsoid center vanishes.

The following experiment is the most spectacular demonstration of the ray walk-off inside a birefringent crystal. A plane-parallel plate cut from a crystal at some angle to the optical axis is illuminated by a parallel beam of naturally polarized light (see Fig. 8) perpendicular to the plate surface. Since the wavevector of the incident wave is orthogonal to the surface, there is no refraction of the wavevector on the plate surface. Both waves (of ordinary and extraordinary polarization) propagate inside the crystal so that their wavevectors are perpendicular to the surface. However, the extraordinary ray is walked-off and its direction of propagation does not coincide with the normal to the surface. The ordinary ray is not walked-off and propagates perpendicular to the surface. There is no refraction at the second surface either. As a result, the rays exiting the crystal are separated and parallel. One can see that the corresponding waves turn out to be linearly polarized in the orthogonal directions.

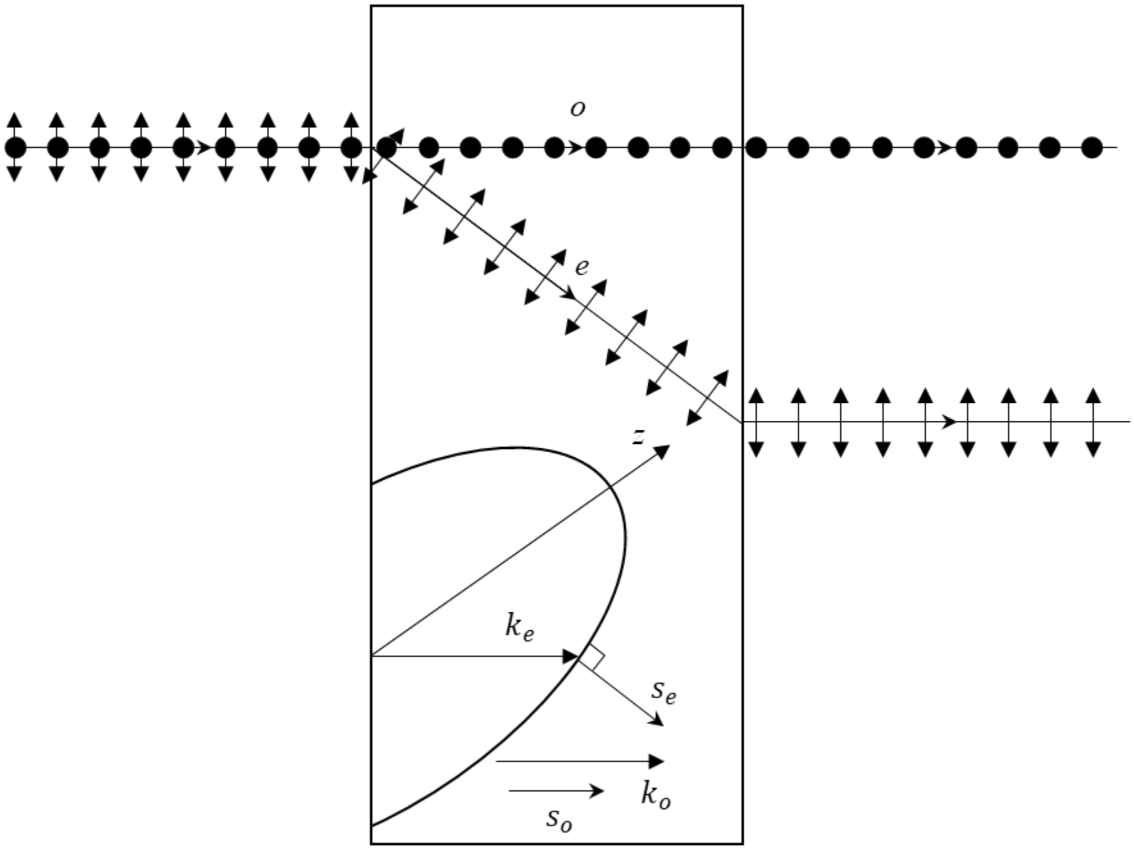


*Fig. 8. Walk-off of rays in uniaxial crystal at normal incidence.*

As an example, consider a classical optical problem. Consider a point drawn on a paper. The paper is then covered by a transparent plate cut from a uniaxial birefringent crystal. The observer eye is at a large enough distance above the point at the vertical drawn from the point. Let us plot the virtual image of this point observed by the eye.

First, consider the rays corresponding to the «*o*»-wave (see Fig. 9). This is no accident that this wave is called ordinary because the problem for this wave is solved as in the case of isotropic medium. According to the problem symmetry the image will be located on the normal to the surface connecting the point being observed and the observer's eye. Rays coming to the observer's eye are almost perpendicular to the plate surface. Their wave vectors are at very small angles to the normal of the crystal surface. Therefore, the wavevectors of these rays inside the crystal are also at small angles to the normal. Let us pick one of the rays going from the point to the eye almost at the right angle to the crystal surface. The refraction angle of this ray (after its exit to the open air) will be greater than the angle of incidence on the surface. The straight line continuing the refracted ray intersects the normal to the surface drawn from the point being observed at some distance from this point. It can be proven that the position of intersection is independent of the angle between the ray and the normal provided the angle remains small. Then the virtual image of the point being observed will be at the point of intersection. It will be located below the refracting crystal surface above the point being observed.

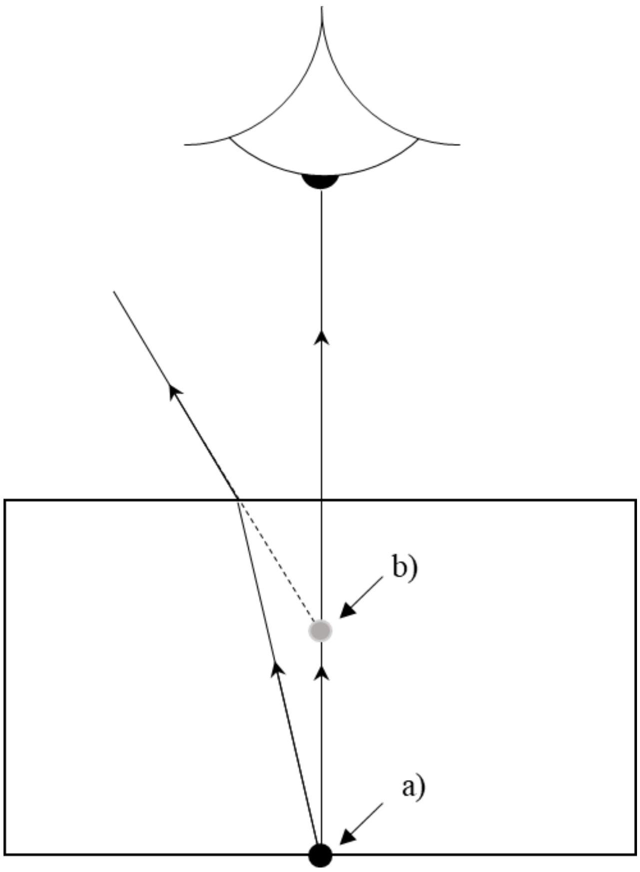


*Fig. 9. Construction of virtual image formed by ordinary rays.*
*a) is location of point-like object; b) is location of its image.*

Now consider formation of a virtual image by the rays of extraordinary wave (see Fig. 10). Let the optical axis of the crystal be rotated by some angle relative to the plate sides. Using symmetry argument one can conclude that the image will lie in the plane formed by the normal to the crystal surface connecting the eye and the point being observed and by the crystal optical axis. Further analysis proceeds for the rays lying in this plane. By repeating the previous arguments one can say that the rays coming to the eye are perpendicular to the surface since the eye is far above the surface. According to the law of refraction the wavevectors of these rays inside the crystal also go at small angles to the normal to the plate surface. Consider two rays inside the plate coming from the point being observed. One ray has the vertical wavevector $\vec{k}_1$, the other ray has the wavevector $\vec{k}_2$ at a small angle to the vertical. The chosen rays will experience a walk-off inside the crystal. Let the vectors in the walk-off directions be $\vec{s}_1$ and $\vec{s}_2$, respectively. The directions of these vectors can be found with the help of the refractive index ellipsoid. After passing the crystal bulk the rays are refracted at the crystal-air interface. The ray with the wavevector $\vec{k}_1$ will not have its wavevector refracted because the wavevector is orthogonal to the interface. Refraction of the ray with the wavevector $\vec{k}_2$ is determined by the refractive index corresponding to the direction of the ray propagation in the crystal bulk. The rays exiting the crystal come to the observer's eye which perceives the virtual image at the point of the ray intersection. This image will be located above the refracting crystal surface above the point being observed and will be shifted to a side from the line connecting the eye and the point being observed.

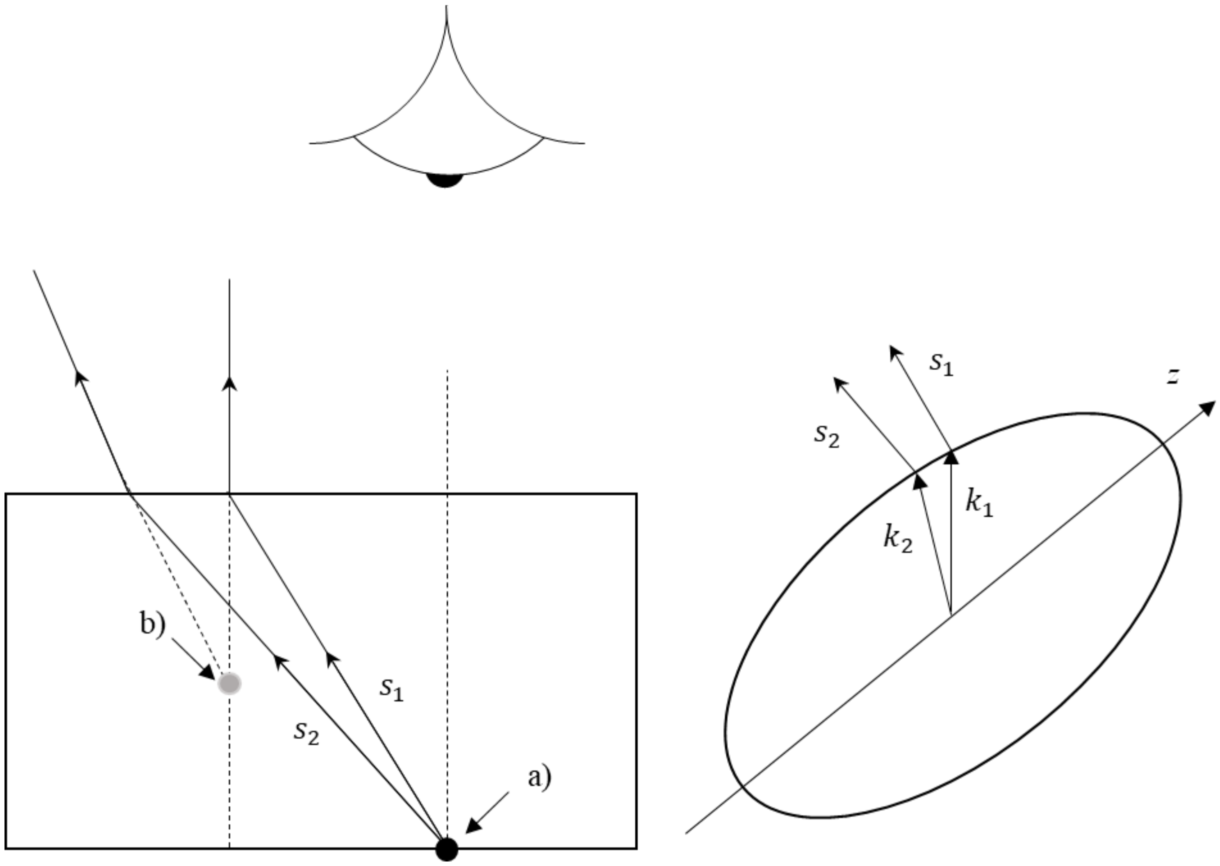


*Fig. 10. Constructing image formed by extraordinary rays.*
*a) is location of point-like object; b) is location of its image.*

Thus, the observer sees two images of the point simultaneously! The first image is formed by ordinary wave and lies below the crystal surface on the normal to the surface connecting the eye and the point being observed. The location depth of the image is due to the refractive index of the ordinary wave for the crystal. The second image is formed by extraordinary rays. It is shifted relative to the first image in the plane passing through the normal to the crystal surface, connecting the eye and the point being observed, and the crystal optical axis. This effect is determined by the ray walk-off inside the crystal. The location of the image is determined both by the refractive index and a change in the ray walk-off caused by a change of the wavevector direction at the point on the refractive index ellipsoid corresponding to the vertical wavevector.

Amplitudes of ordinary and extraordinary waves propagating in the crystal depend on the amplitude of the incident wave, the direction of its propagation and on its polarization as well. It could happen that only a single wave will propagate in the crystal. For instance, suppose that a linearly polarized plane wave is incident on the crystal surface. Suppose that direction of oscillation of electric field of this wave is perpendicular to the polarization plane of one of the eigenwaves («o» - wave or «e» - wave) of the crystal. In this case the amplitude of the corresponding wave in the crystal will vanish, which is equivalent to the absence of this wave. If so, only that wave will propagate in the crystal for which the component of the electric field vector of the incident wave on the polarization plane of the crystal eigenwaves is nonzero.

Birefringence is widely used in science and technology applications. In particular, such crystals are used in the devices designed to control polarization of radiation. Consider one of these devices.

Suppose a uniaxial crystal of a thickness $d$ is cut as a plate with the sides parallel to the crystal optical axis. Assume that a linearly polarized radiation is incident on the plate perpendicular to its surface. Ordinary and extraordinary waves will arise inside the crystal and propagate perpendicular to the crystal surface. The refractive indices for these waves are different. Unlike at the wave entry on the plate, where the fields in «*o*» and «*e*» waves oscillate in phase, the oscillations exiting the plate will have a phase shift. This phase shift is easy to express in terms of the difference of magnitudes of wavevectors of «*o*» and «*e*» waves and the plate thickness:

$$\Delta\varphi = (k_e - k_o)d = \frac{\omega}{\mathrm{c}}(n_e - n_o)d. \quad (17)$$

The ordinary and extraordinary waves after passing through the crystal are linearly polarized in the orthogonal planes. Adding the vectors of electric fields of the waves gives the common ray field vector which endpoint will draw an ellipse. Therefore, it is said that the light polarization becomes elliptical. The shape of this ellipse depends on the phase shift between the waves of different polarization. For the phase shift $\Delta\varphi = \pi N$, where $N$ is an integer the ellipse will be reduced to a straight line, which corresponds to linear polarization. The polarization remains the same for even $N$ and rotates by an angle relative to the polarization of the incident wave if $N$ is odd.

If $N = 1$, the optical path difference equals one half the wavelength of the radiation in vacuum. In this case, it is said that the crystal is a half-wave plate. Consider the principle of half-wave plate operation in some detail. Suppose a linearly polarized light is incident on such a plate at some angle $\alpha$ to its optical axis. Two waves of orthogonal polarizations will propagate inside the plate. We will represent graphically a decomposition in two linear polarizations as a superposition of two orthogonal vectors of electric field along the plate optical axis (the vertical in Fig. 11) and perpendicular to the optical axis (the horizontal in Fig. 11) and oscillating at a frequency $\omega$. Vectors $\vec{E}_o$ and $\vec{E}_e$ oscillate in phase when entering the plate, so their sum $\vec{E}_{in}$ also oscillates with the same phase along the direction of polarization. Upon exiting the plate, the phase shift between electric oscillations of different polarization becomes equal $\pi$. Assume, for instance, that before a ray entered the plate the electric field vector of extraordinary wave was pointing upward when the vector of ordinary wave was pointing to the right. Then, after passing the plate, the field vector of extraordinary wave will point downward when the vector of ordinary wave points to the right. The electric field vector of the wave coming out of the plate, $\vec{E}_{out}$, is the sum of the field vectors of ordinary and extraordinary waves. The vector $\vec{E}_{out}$ oscillates in phase with the incident vector $\vec{E}_{in}$ and is rotated with respect to it by an angle of $\pi - 2\alpha$. Thus, the wave polarization remains linear but now it is rotated clockwise by the angle $\pi - 2\alpha$.

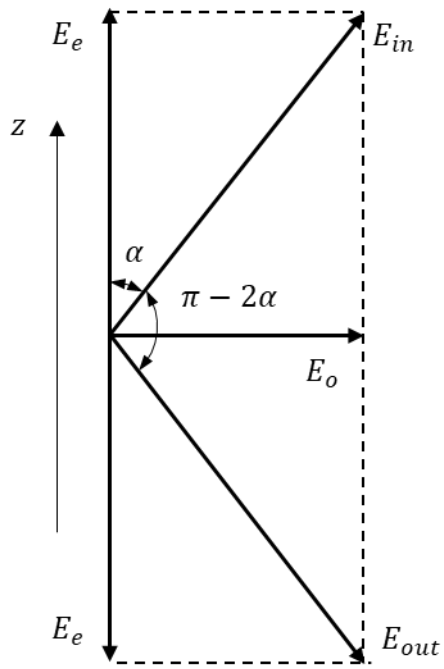


*Fig. 11. Rotation of linear polarization by means of half-wave plate.*

Let us take a polarizer plate of arbitrary thickness and place another polarizer crosswise behind it, so that their transmission planes are perpendicular. Such a polarizer is called the analyzer. If a plate does not change the polarization plane of incoming radiation (a plate with an optical path difference of an integer number of wavelengths), the analyzer does not transmit the radiation. If the optical path difference in the plate is not equal to an integer number of wavelengths, the outgoing radiation will have different polarization with respect to the incoming one, e.g. it could become elliptically polarized. In this case the analyzer partially transmits the radiation.

A separate case to consider is when the polarization plane of incident radiation and the optical axis of a half-wave plate are at 45 degrees. In this case the polarization of the wave exiting the plate rotates at 90 degrees with respect to the incident wave and the radiation passes through the analyzer without a loss.