6th Olympiad of Metropolises

Mathematics · Day 2

Problem 4. Six real numbers $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ are given. For each triplet of distinct numbers of those six Vitya calculated their sum. It turned out that the 20 sums are pairwise distinct; denote those sums by

$$s_1 < s_2 < s_3 < \cdots < s_{19} < s_{20}\,.$$

It is known that $x_2 + x_3 + x_4 = s_{11}$, $x_2 + x_3 + x_6 = s_{15}$ and $x_1 + x_2 + x_6 = s_m$. Find all possible values of m.

Problem 5. There is a safe that can be opened by entering a *secret code* consisting of n digits, each of them is 0 or 1. Initially, n zeros were entered, and the safe is closed (so, all zeros is not the secret code).

In one attempt, you can enter an arbitrary sequence of n digits, each of them is 0 or 1. If the entered sequence matches the secret code, the safe will open. If the entered sequence matches the secret code in more positions than the previously entered sequence, you will hear a click. In any other cases the safe will remain locked and there will be no click.

Find the smallest number of attempts that is sufficient to open the safe in all cases.

Problem 6. Let $ABCD$ be a tetrahedron and suppose that M is a point inside it such that $\angle MAD = \angle MBC$ and $\angle MDB = \angle MCA$. Prove that

$$MA \cdot MB + MC \cdot MD < \max(AD \cdot BC, AC \cdot BD)\,.$$