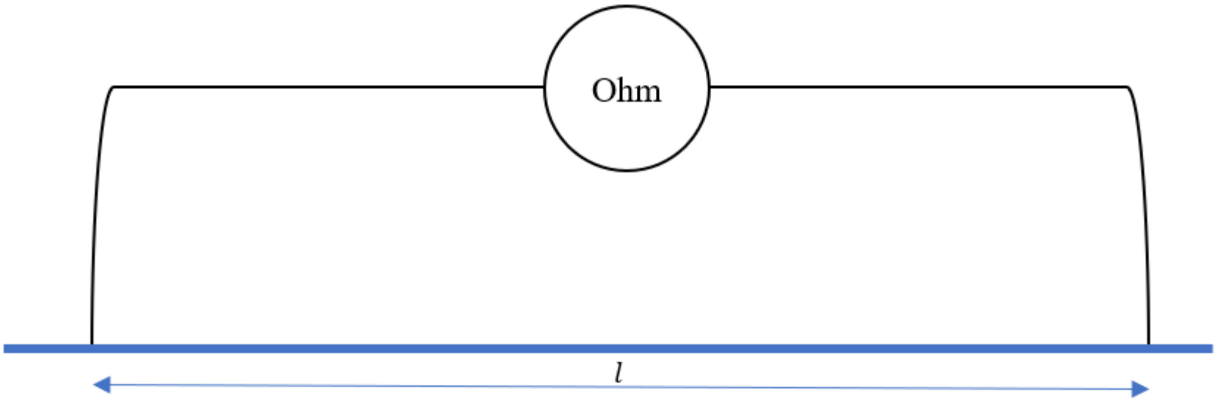
Answer sheet

Part 1. Measurement of the dependence of the resistance of a part of the wire on its length.

1.1. Record the measurement results from the video instruction in the table.

R, Ohm	l, cm
21.0	14.4
23.4	16.2
26.3	18.5
29.1	20.6
32.3	22.9
35.5	25.2
38.7	27.7
42.2	30.2
46.8	33.5
51.8	37.2
56.0	40.4
65.7	47.7
71.3	51.9
80.0	58.5
83.6	61.9

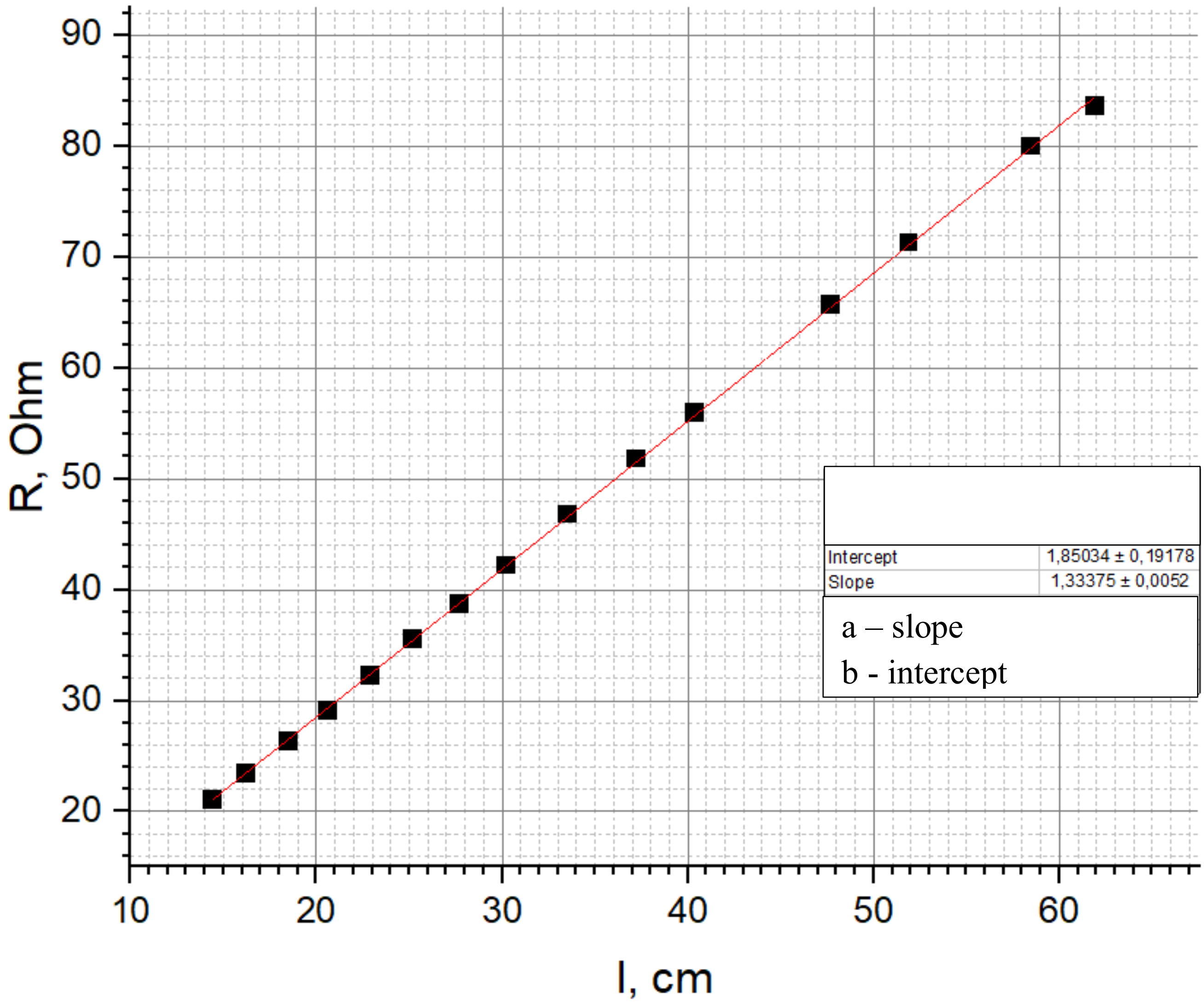
1.2. Sketch the measurement scheme that was used in the experiment.



1.3. Write down a theoretical formula describing the relationship between the resistance (R) of the nichrome wire with its length (l), cross-sectional area (S) and its resistivity (ρ).

$$R = \frac{\rho l}{S}$$

1.4. Build a graph of the dependence of the resistance of the nichrome wire segment on its length $R(l)$.



Fit the obtained points with a linear function $R = ax + b$ and write down the values of the approximation parameters.

$$a = 1.33 \pm 0.05 \ \frac{Ohm}{cm}$$

$$b = 1.8 \pm 0.4 \ Ohm$$

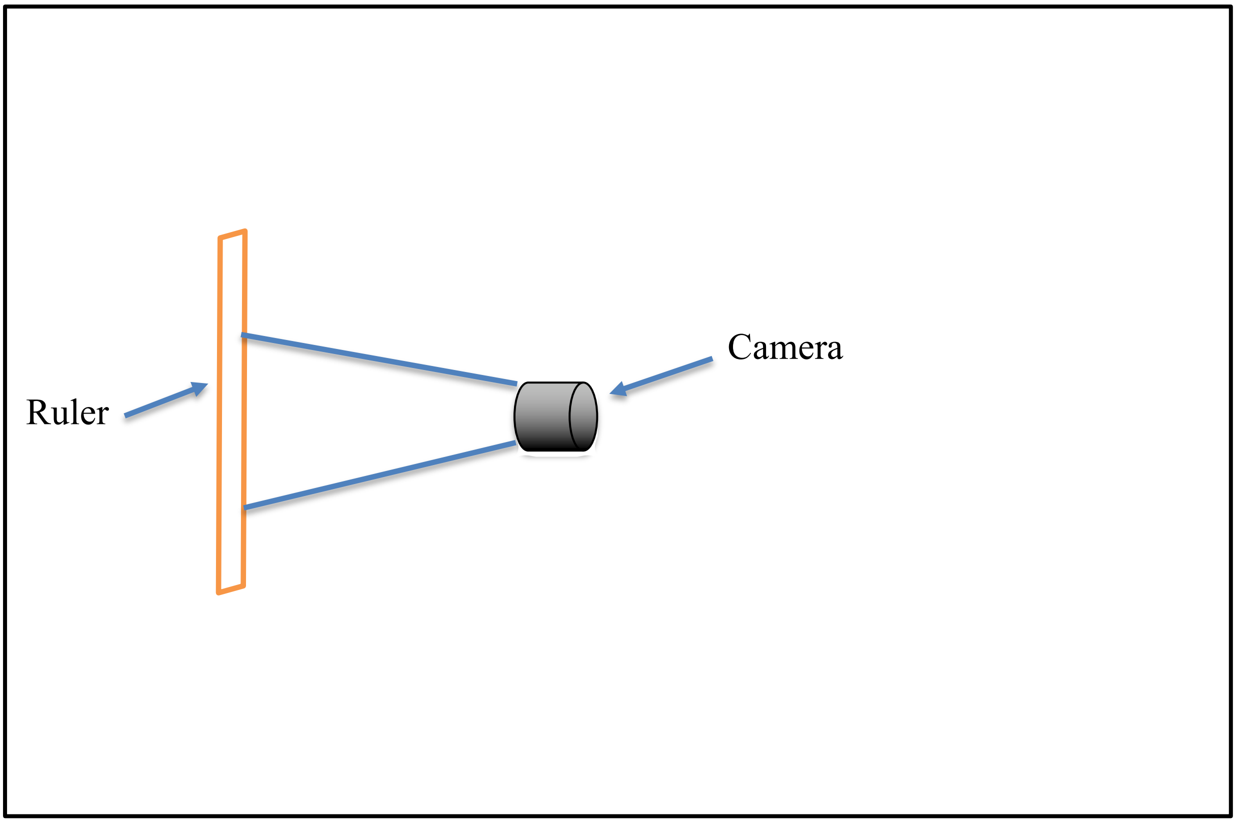
Give a brief interpretation of the approximation parameters a and b.

$$a = \frac{\rho}{S}$$

b – contacts and connecting wires resistance

Part 2. Preparing a computer program for work.

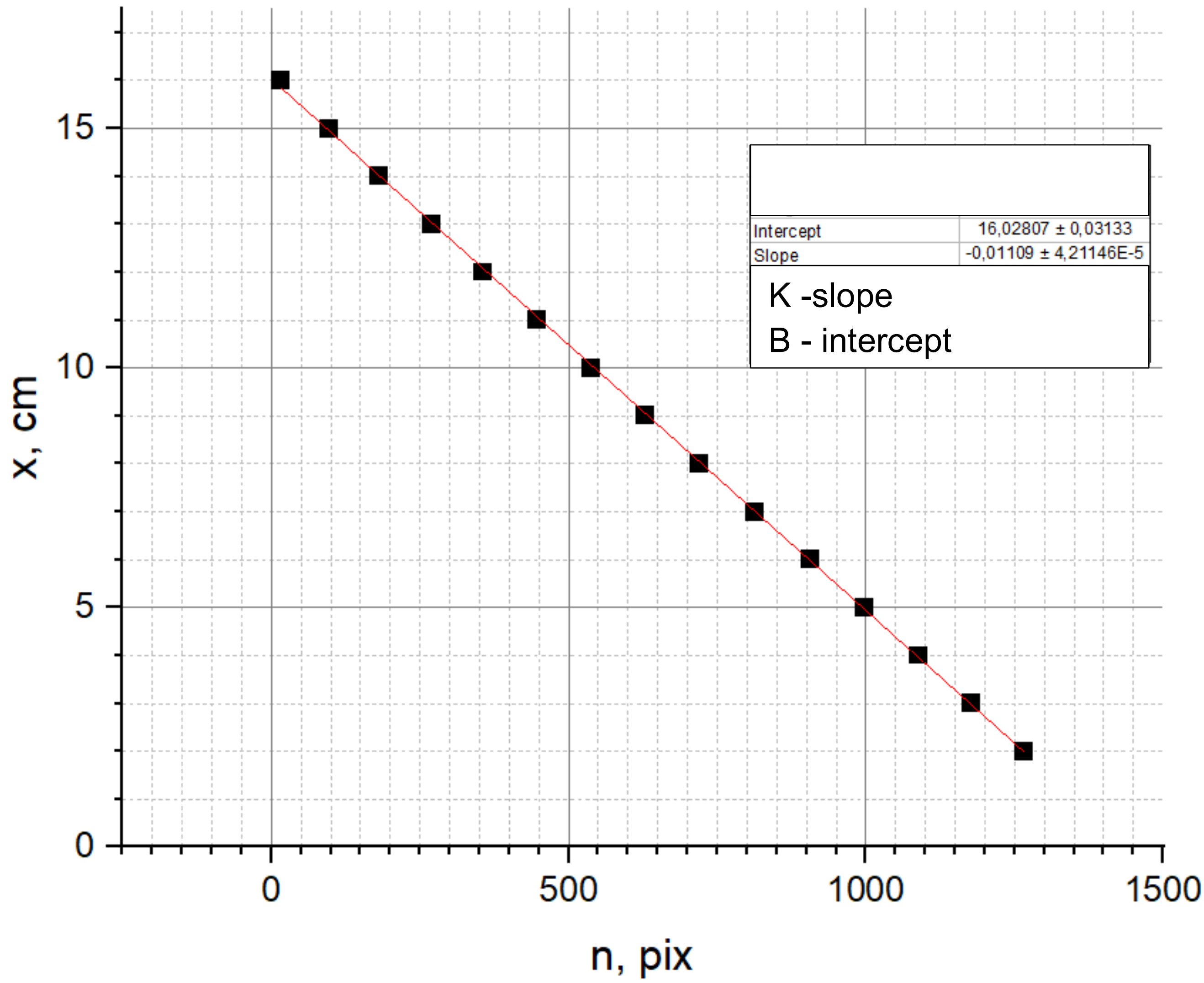
2.1. Draw a diagram of the set-up (specifically, the location of the camera and the ruler), which is used to obtain an image of the ruler for further calibration.



2.2. Process the image of the ruler in the program. Then enter the obtained data of the dependence of the coordinates on the ruler (x) in centimeters on the pixel number (n) in the image into the table.

x, cm	n, pix
16	16
15	98
14	182
13	269
12	357
11	447
10	537
9	629
8	721
7	814
6	906
5	998
4	1089
3	1178
2	1266

2.3. Plot the value of x versus the value of n (the dependence $x(n)$). Draw a straight line ($x = Kn + b$) through the obtained experimental points, which best describes the dependence and then determine its slope K.

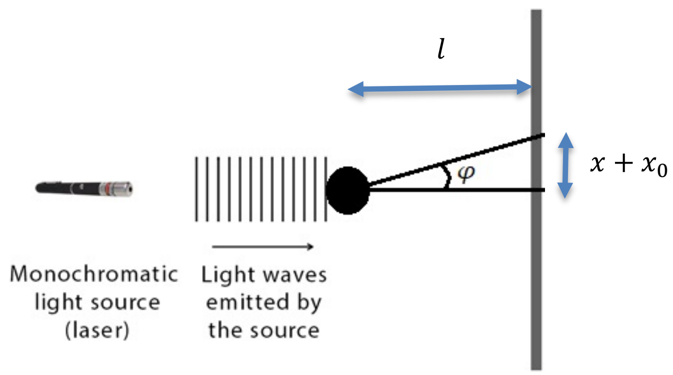


2.4. Write down the value of the angular slope coefficient K for the resulting straight line, and also enter it into the program so that this coefficient can be used later to convert the coordinate in pixels into a coordinate in centimeters.

$$K = -0.011 \text{ or } +0.011 \ \frac{cm}{pix} \quad \pm 0.001$$

Part 3. Measurement of the diameter of a nichrome wire in an unstressed state using a diffraction pattern.

3.1. Draw a scheme of the setup for obtaining and studying the diffraction pattern. On the drawn scheme, mark with letters the required distances measured in the experiment.



3.2. Write down the measured distance between the screen and the wire.

$l = 52.5$ cm ± 0.1 cm

3.3. Process the image of the diffraction pattern, obtained in the unstressed state of the wire. Using the obtained graph of the light intensity versus the coordinate along the ruler, measure the dependence of the coordinate of the diffraction minimum (x) on its number (N), including minima with negative ordinal numbers.
Record the obtained experimental points in the table.

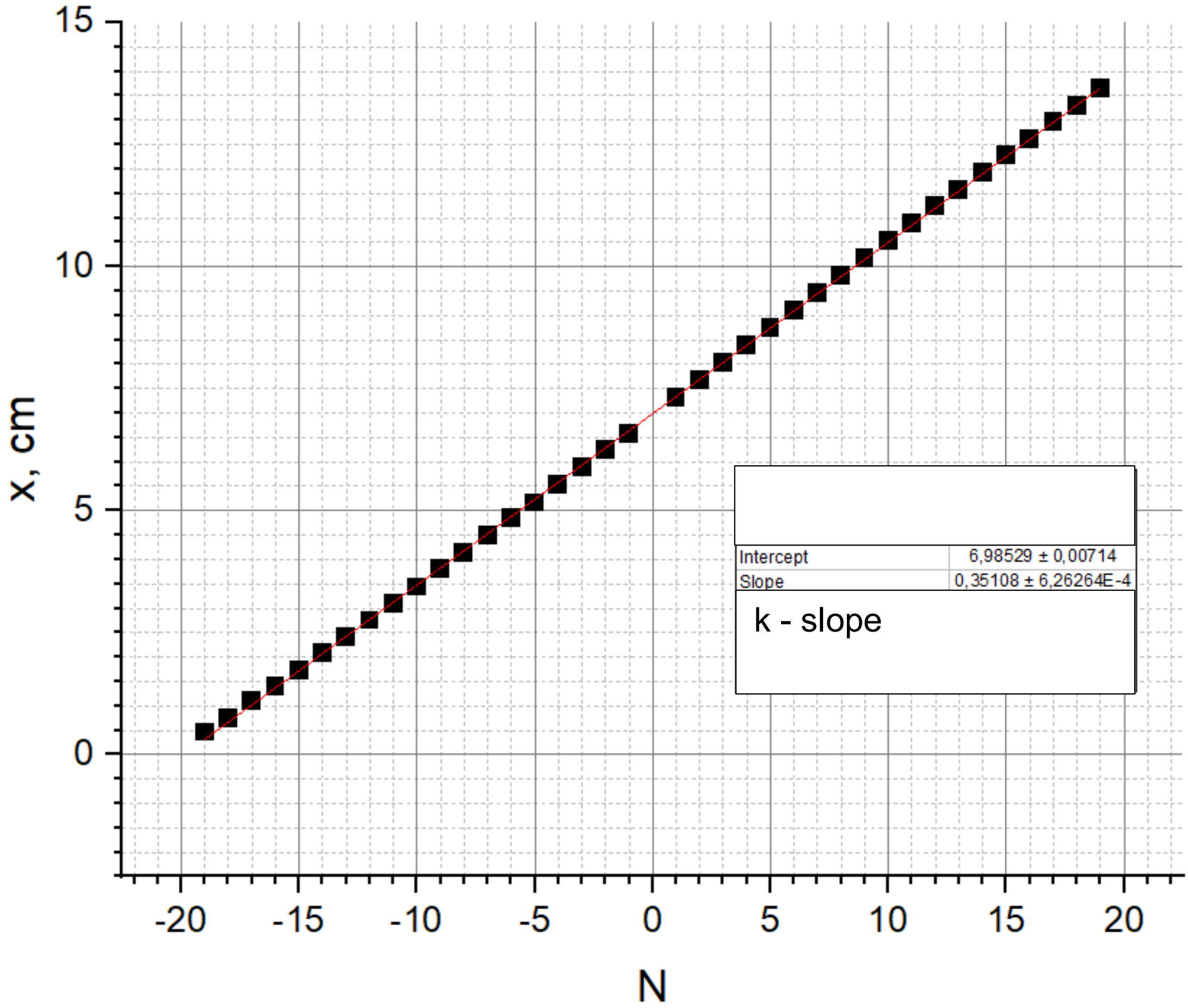
x, cm	N
7.304	1
7.667	2
8.030	3
8.382	4
8.734	5
9.097	6

9.449	7
9.812	8
10.164	9
10.527	10
10.890	11
11.242	12
11.572	13
11.935	14
12.276	15
12.617	16
12.969	17
13.299	18
13.651	19
6.578	-1
6.237	-2
5.885	-3
5.533	-4
5.159	-5
4.840	-6
4.477	-7
4.125	-8
3.795	-9
3.432	-10
3.102	-11
2.750	-12
2.420	-13
2.079	-14
1.716	-15
1.397	-16
1.100	-17
0.737	-18
0.462	-19

3.4. Write down the formula describing the relationship between the coordinates of the minima (x) and their numbers (N). In this case, consider that the light propagates at small angles to its initial direction of propagation.

$$x = x_0 + \frac{n\lambda l}{d}$$

3.5. Plot the measured dependence $x(N)$.



Determine to what minimum number the graph can be considered linear. Find the slope (k) of a linear portion of the graph.

The graph deviates from linear dependence at $N < -15$. *It should also be noted that in the program on the graph of pixel intensity from the coordinate, the extreme minima are blurred (2 cm from each edge), which does not allow for guaranteed determination of their coordinate, so we should not carry out further measurements in these areas.*

$$k = (0{,}35 \pm 0{,}01)\ \text{cm}$$

3.6. Calculate the diameter (d_0) of the unstressed nichrome wire using the obtained slope of the graph. Write down the result:

$$d_0 = \frac{\lambda l}{k} = \frac{650 \cdot 10^{-9} \cdot 52.5 \cdot 10^{-2}}{0.351 \cdot 10^{-2}} = 97 \pm 4\ \mu m$$

Part 4. Investigation of the parameters of nichrome wire when it is stretched.

4.1. Record the values of the coordinates of the contacts for the investigated section of the wire (L_1 and L_2), and the readings of the voltmeter (U) and ammeter (I) in the table.

№	L_1, cm	L_2, cm	I, mA	U, V	Δx, cm
1	63.9	11.1	53.7	3.93	7.486
2	64.5	11.1	52.6	3.93	7.496
3	65.2	11.2	51.5	3.94	7.497
4	65.9	11.2	50.4	3.94	7.497
5	66.2	11.3	49.4	3.94	7.530
6	67.0	11.4	48.5	3.95	7.574
7	67.7	11.4	47.6	3.95	7.608
8	68.5	11.5	46.7	3.95	7.630
9	69.2	11.7	45.9	3.95	7.663
10	70.0	11.8	45.0	3.96	7.696
11	70.7	11.9	44.1	3.96	7.741
12	71.1	12.0	43.3	3.96	7.763
13	71.8	12.1	42.5	3.96	7.807
14	72.6	12.2	41.8	3.97	7.818
15	73.1	12.2	41.1	3.97	7.874
16	73.8	12.2	40.3	3.97	7.907
17	74.6	12.2	39.5	3.97	7.940
18	75.4	12.2	38.8	3.98	7.963

4.2. Write down the numbers of the selected minima, the distance between which will be measured.

$n_1 = -11$ $n_2 = 10$

For each diffraction pattern, measure the distance between the selected minima and enter it in the appropriate column of the table in paragraph 4.1.

4.3. Write down the following formulas:

a) the formula for determining the length of the investigated section of the wire (L),

b) the formula for calculating the electrical resistance of the investigated section of the wire (R),

c) expression for calculating the diameter of the wire (d).

This is necessary for the subsequent calculation of all required parameters based on the data from the table in paragraph 4.1.

$$L = L_2 - L_1$$

$$R = \frac{U}{I}$$

$$d = \frac{\lambda|n_2 - n_1|}{\varphi} = \frac{\lambda l|n_2 - n_1|}{\Delta x}$$

4.4. Write down in the table the length of the selected wire section (L), its electrical resistance (R) and wire diameter (d) values for each stretching step.

№	L, cm	R, Ohm	d, μm	ln(L/cm)	$ln(R/\text{Ohm})$	$ln(d/\mu\text{m})$
1	52.8	73,2	95.73	3.967	4.293	4.5615
2	53.4	74,7	95.60	3.978	4.314	4.5602
3	54.0	76,5	95.59	3.989	4.337	4.5600
4	54.7	78,2	95.59	4.002	4.359	4.5600
5	54.9	79,8	95.17	4.006	4.379	4.5557
6	55.6	81,4	94.62	4.018	4.400	4.5498
7	56.3	83,0	94.19	4.031	4.419	4.5454
8	57.0	84,6	93.92	4.043	4.438	4.5425
9	57.5	86,1	93.52	4.052	4.455	4.5381
10	58.2	88,0	93.12	4.064	4.477	4.5339
11	58.8	89,8	92.58	4.074	4.498	4.5280
12	59.1	91,5	92.31	4.079	4.516	4.5252
13	59.7	93,2	91.79	4.089	4.534	4.5195
14	60.4	95,0	91.66	4.101	4.554	4.5181
15	60.9	96,6	91.01	4.109	4.571	4.5110
16	61.6	98,5	90.63	4.121	4.590	4.5068
17	62.4	100,5	90.26	4.134	4.610	4.5026
18	63.2	102,6	89.99	4.146	4.631	4.4997

4.5. Using the expression for Poisson's ratio, obtain the relationship between the logarithm of the diameter and the logarithm of the length of the wire section. This can be done using integration. Write down the resulting formula.

$$\mu = -\frac{\Delta d}{d}\frac{L}{\Delta L}$$

$$\mu\frac{\Delta L}{L} = -\frac{\Delta d}{d}$$

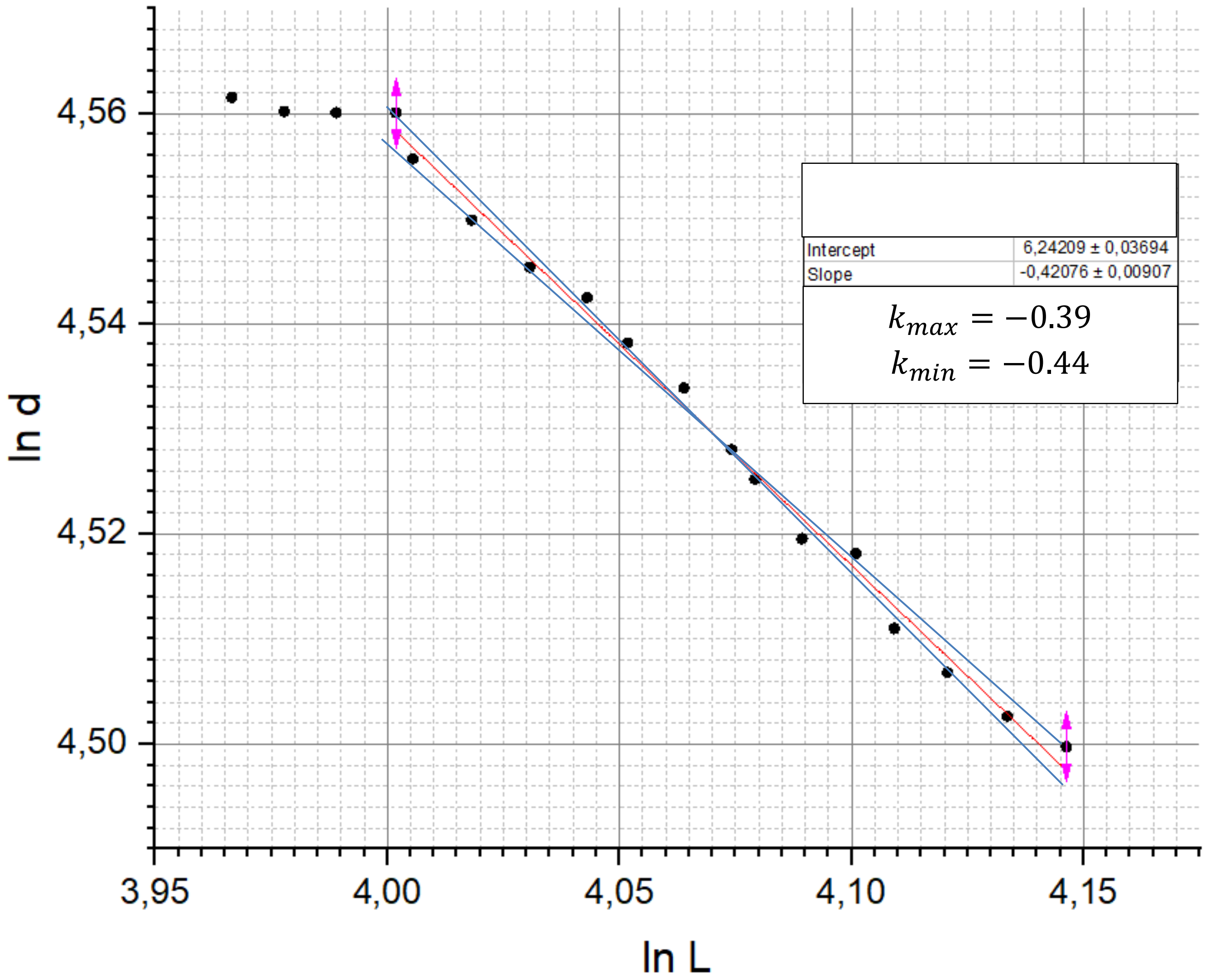
$$\int \mu\frac{\Delta L}{L} = \int -\frac{\Delta d}{d}$$

$$\mu \ln\frac{L}{L_0} = -\ln\frac{d}{d_0}$$

$$\ln d = \ln d_0 + \mu \ln L_0 - \mu \ln L$$

Calculate the corresponding values of the logarithms of these quantities and write down the values in the table in paragraph 4.4.

4.6. Plot the dependence of the logarithm of the wire diameter on the logarithm of its length ($ln\ d(ln\ L)$).



Determine the Poisson's ratio of the wire using the resulting graph.
Write down the resulting Poisson's ratio (μ):

$\mu = 0.42 \pm 0.04$

4.7. Write down the derivation of the formula for the dependence of the logarithm of the resistance ($ln\ R$) of the wire section on the logarithm of its length ($ln\ L$), assuming the resistivity of the wire to be constant.

$$R = \frac{\rho l}{S}$$

$$\ln R = \ln \rho + \ln l - \ln \frac{\pi}{4} - 2 \ln d$$

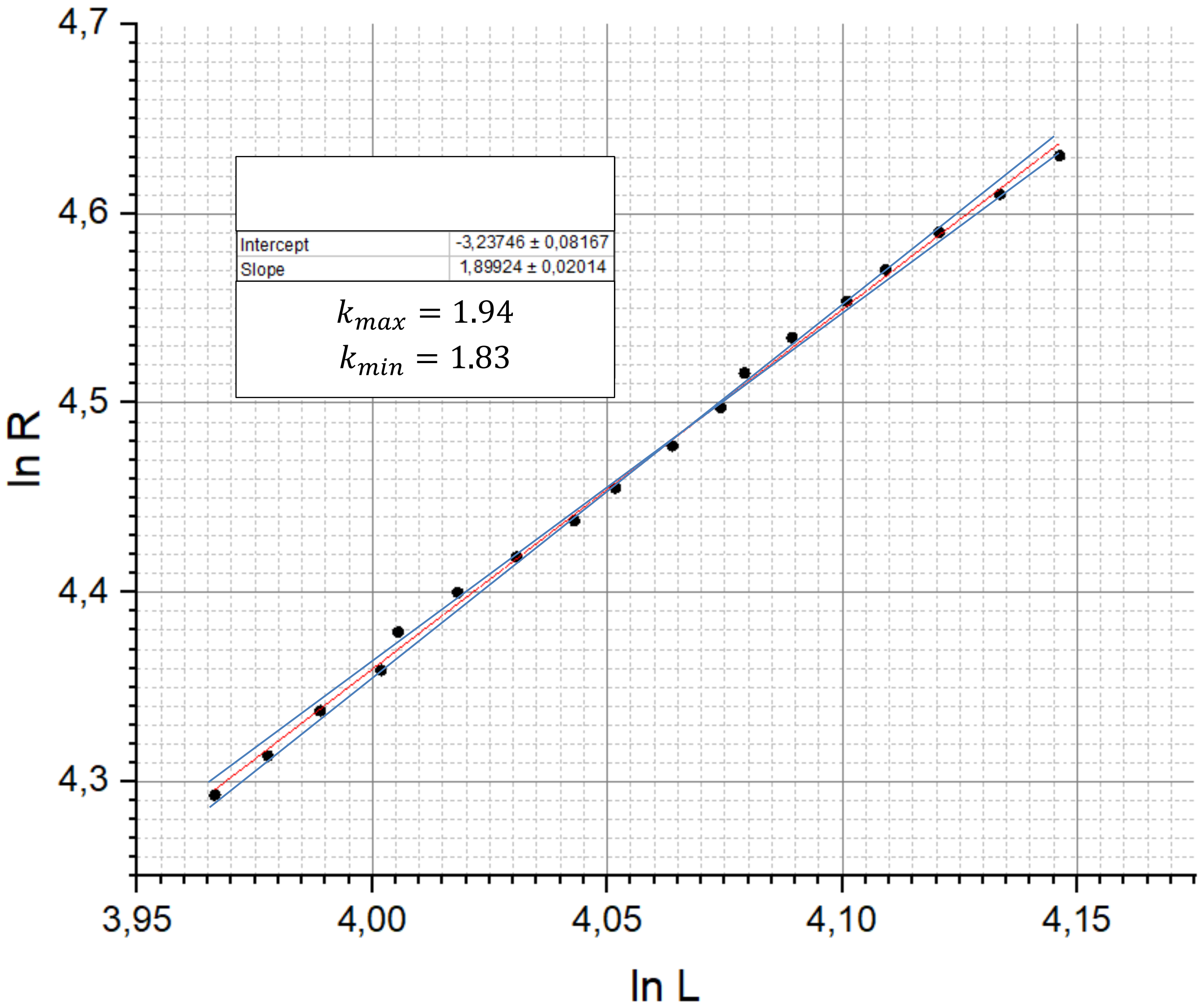
$$\ln R = \ln \rho + \ln l - \ln \frac{\pi}{4} - 2 \ln d_0 - 2\mu \ln l_0 + 2\mu \ln l$$

$$\ln R = const + (1 + 2\mu) \ln l$$

$$const = \ln \frac{4\rho}{\pi d_0^2 l_0^{2\mu}}$$

Write down the values of the logarithms of the resistance of the wire section at each stage of stretching in the table in paragraph 4.3.

4.8. Plot the dependence of the logarithm of the resistance of the wire section on the logarithm of its length $lnR(lnL)$.



Determine the slope (k) of the resulting graph and write it down:

$$k = 1.9 \pm 0.1$$

4.9. Based on a comparison of the obtained values of the slopes for the graphs from paragraphs 4.6 and 4.8, draw a conclusion about how the resistivity changes with increasing wire length.

Underline your choice:

decreases | does not change | increases

and provide a brief explanation of your choice:

From the dependence of the diameter on the length: $\mu = 0.42 \pm 0.02$

From the dependence of resistance on length: $k = 1.90 \pm 0.05 = 1 + 2\mu$

$$\mu = \frac{1.9 - 1}{2} = 0.45 \pm 0.03$$

The values almost coincide within the margin of error, hence the assumption that resistivity does not depend on elongation was correct.

Part 5. Determination of the thermal resistance coefficient.

5.1. Record the value of the voltage on the wire (U_t) at room temperature:

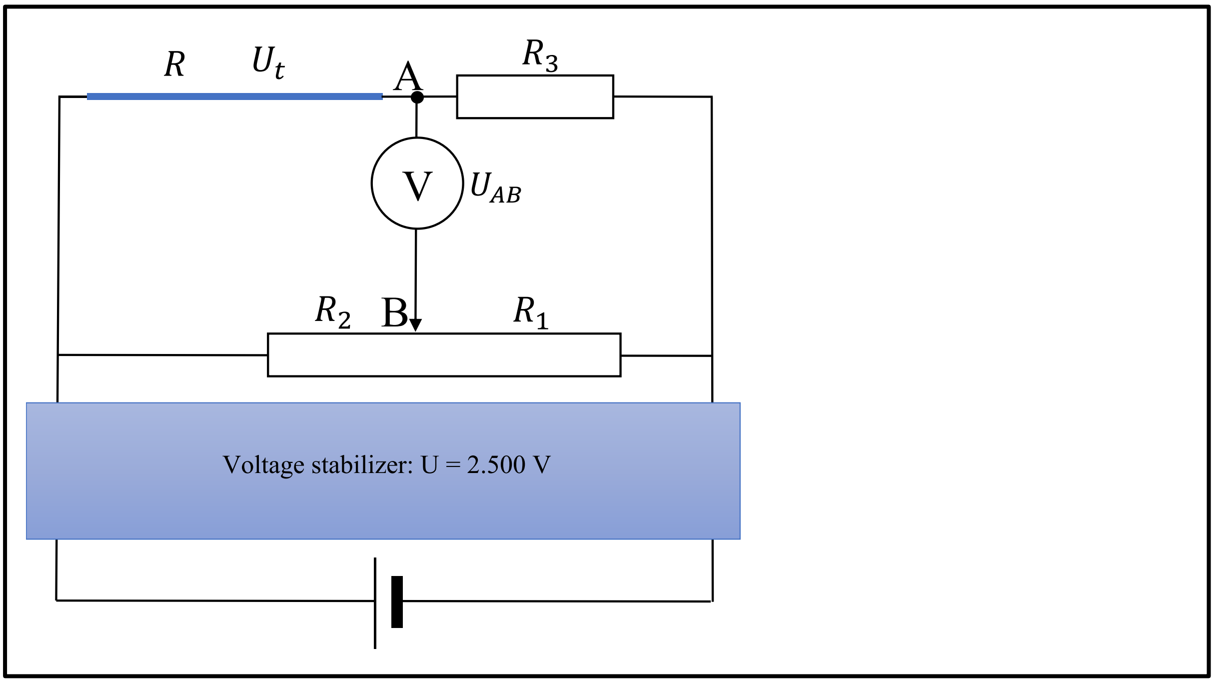
$U_t = 1256$ mV

5.2. Record the voltage values obtained in the experiment between points A and B (U_{AB}) and the corresponding temperature values (t) in the table:

U_{AB}, mV	t, °C
3,6	45.1
5,6	52.7
6,9	56.6
8,5	61.8
9.6	65.1
10.2	66.8
11.2	69.8
11.6	70.6
11.9	71.4
12.1	71.8
12.2	71.7
12.1	71.1
11.9	70.0
11.6	69.0
11.4	67.9
11.1	67.0
10.9	66.0
10.3	63.4
9.9	61.7
9.4	59.8

9.2	58.7
8.9	57.3
8.7	56.7
8.3	55.0
7.5	52.1
7.0	50.1
6.7	48.9
6.4	47.5
6.2	47.0
6.0	46.0
5.7	45.0

5.3. Draw an electrical diagram and mark with letters the parameters necessary for further calculations.



5.4.

a) Write the derivation of the formula expressing the change in the resistance of the wire (ΔR) through its resistance at room temperature (R_0), the voltage of the power source ($U = 2500$ mV) and the voltage (U_t) on the wire at room temperature. Consider the change in wire resistance small.

$$U_{AB} = I_3R - I_1R_2 = \frac{U}{R_3 + R}R - \frac{U}{R_1 + R_2}R_2$$

$$\frac{dU_{AB}}{dR} = U\frac{R_3 + R - R}{(R_3 + R)^2} = \frac{UR_3}{(R_3 + R)^2}$$

$$dU_{AB} = \frac{UR_3}{(R_3 + R)^2}dR \approx \frac{UR_3}{(R_3 + R_0)^2}dR$$

(R_0 - wire resistance at room temperature)

$$dU_{AB} = U\frac{R_3}{R_3 + R_0} \cdot \frac{R_0}{R_3 + R_0} \cdot \frac{dR}{R_0}$$

$$\frac{U}{R_3 + R_0} = \frac{U - U_t}{R_3} = \frac{U_t}{R_0} \rightarrow$$

$$dU_{AB} = U\frac{U - U_t}{U} \cdot \frac{U_t}{U} \cdot \frac{dR}{R_0} = \frac{U_t(U - U_t)}{U} \cdot \frac{dR}{R_0}$$

$$\Delta U_{AB} = \frac{U_t(U - U_t)}{U} \cdot \frac{\Delta R}{R_0}$$

$$\Delta U_{AB} = U_{AB} - 0 = U_{AB}$$

$$U_{AB} = \frac{U_t(U - U_t)}{U} \cdot \frac{\Delta R}{R_0}$$

$$\Delta R = \frac{R_0 U_{AB} U}{U_t(U - U_t)}$$

If consider that the resistance in each branch of the bridge circuit are equal at the beginning, then we can get the result:

$$dU_{AB} = \frac{U}{4} \cdot \frac{dR}{R_0}$$

$$\Delta R = \frac{4 \cdot R_0 U_{AB}}{U}$$

b) Write the derivation of the formula expressing the voltage between points A and B (U_{AB}) through temperature (t), supply voltage (U), voltage (U_t) on the wire at room temperature and the thermal coefficient of resistance of the wire (α).

$$\begin{cases} \alpha = \frac{dR}{R_0}\frac{1}{dt} \rightarrow \alpha = \frac{\Delta R}{R_0}\frac{1}{\Delta t} \\ dU_{AB} = \frac{U_t(U - U_t)}{U} \cdot \frac{dR}{R_0} \end{cases}$$

$$dU_{AB} = U_{AB} = \frac{U_t(U - U_t)}{U} \cdot \alpha \Delta t$$

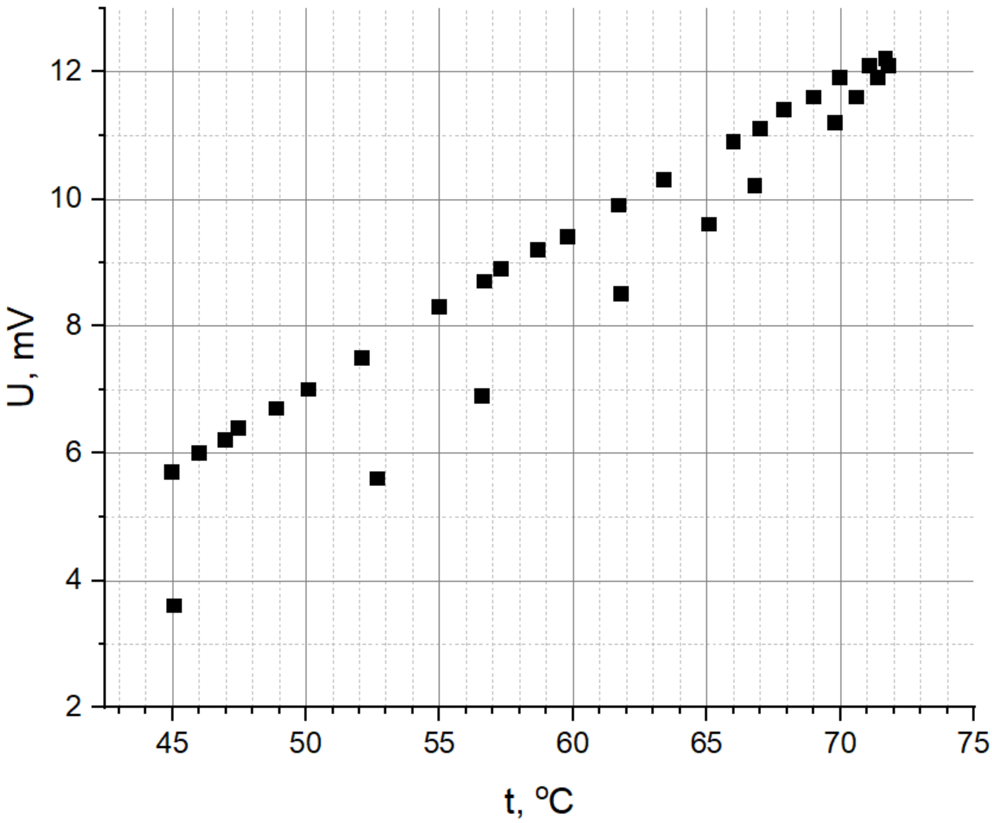
$$U_{AB} = \frac{U_t(U - U_t)}{U} \cdot \alpha \Delta t = \frac{U_t(U - U_t)}{U} \cdot \alpha (t - t_0)$$

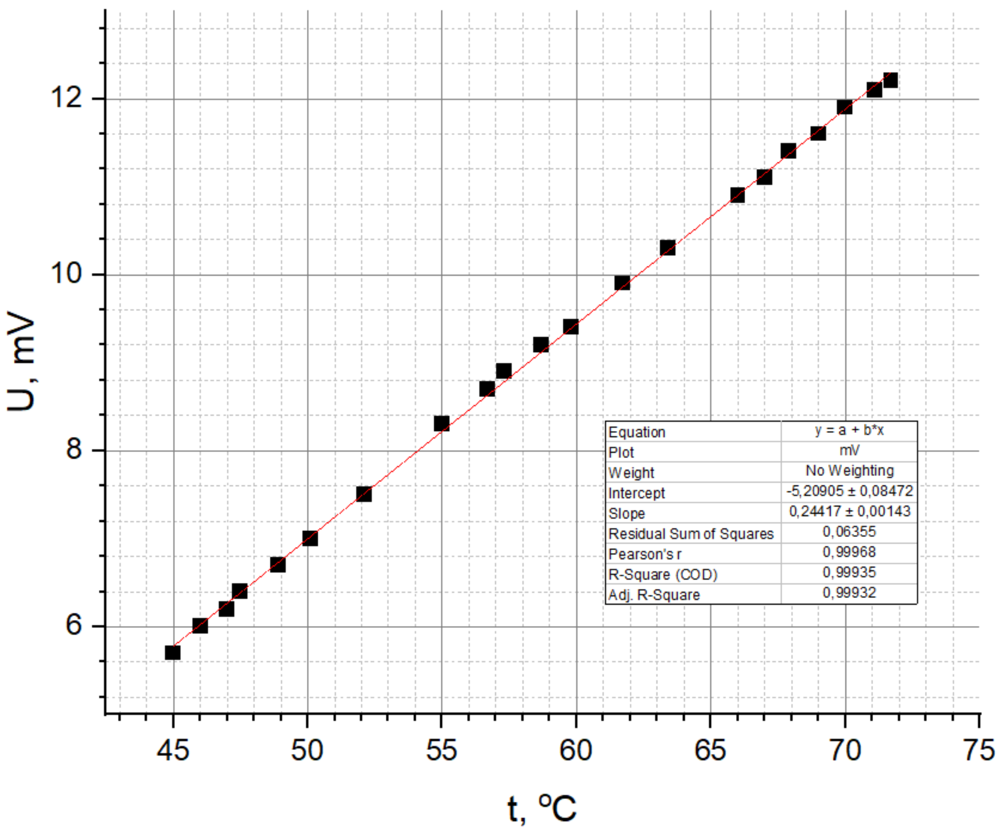
Write down the final formulas:

$$\Delta R = \frac{R_0 U_{AB} U}{U_t(U - U_t)}$$

$$U_{AB} = \frac{U_t(U - U_t)}{U} \cdot \alpha \Delta t = \frac{U_t(U - U_t)}{U} \cdot \alpha (t - t_0)$$

5.5. Plot the dependence of the voltage between points A and B from temperature ($U_{AB}(t)$).





Answer what part of the graph can be considered linear and taken into account when determining the temperature coefficient of resistance.

The lower curve on the graph corresponds to the gradual heating of the wire and the temperature sensor for pouring hot water. This process is non-equilibrium, so it should not be taken into account when calculating the temperature coefficient of resistance.

Using the formula obtained in paragraph 5.4 b), determine from the graph the temperature coefficient of resistance of the wire material in the temperature range you selected.

Write down the obtained coefficient value (α) and the selected temperature range (t):

$$\alpha = \frac{0.244 \cdot U}{U_t(U - U_t)} = \frac{0.244 \cdot 2500}{1256 \cdot (2500 - 1256)} = (0.39 \pm 0.04) \cdot 10^{-3} \frac{1}{°C}$$

$$t \in [45, 70]°C$$

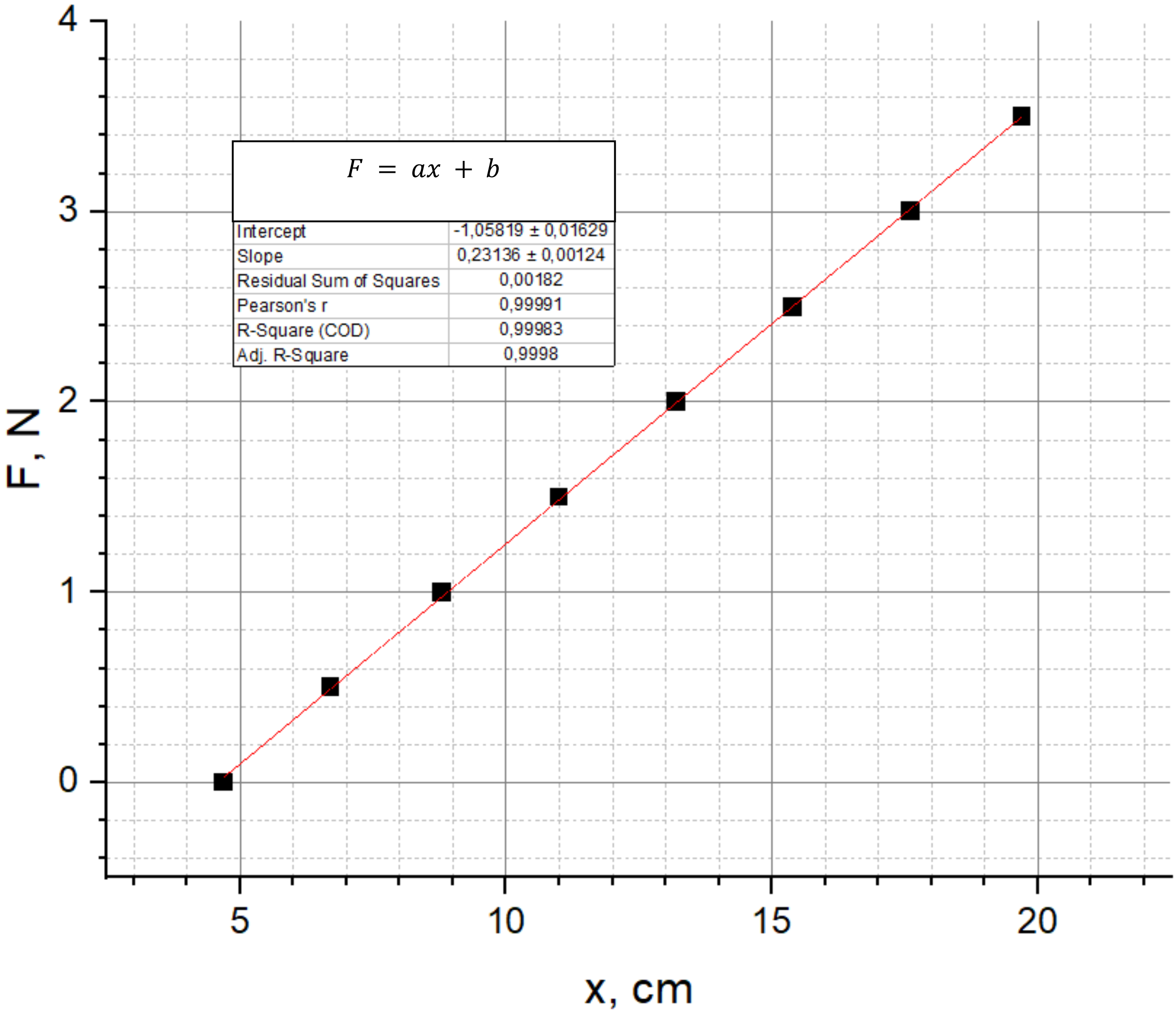
Part 6. Investigation of elastic deformations of the wire.

6.1. Enter the data of the preliminary experiment in the table, namely, the coordinates of the spring markers (x_1 and x_2) and the mass of the load (m) on the spring.

x_1, cm	x_2, cm	m, g	x, cm	F, N
0.6	5.3	0	4.7	0.0
0.6	7.3	50	6.7	0.5
0.7	9.5	100	8.8	1.0
0.7	11.7	150	11.0	1.5
0.8	14.0	200	13.2	2.0
0.8	16.2	250	15.4	2.5
0.9	18.5	300	17.6	3.0
0.9	20.6	350	19.7	3.5

From the measured data, calculate the spring length (x) and the spring load force (F) for each stretch step. Enter the obtained values into the same table.

6.2. Plot the dependence of the spring load force on the spring length $F\ (x)$.



Fit the graph with a linear function ($F = ax + b$) and write down the values of the approximation coefficients.

$$a = 0.23 \pm 0.01 \frac{N}{cm}$$

$$b = -1.06 \pm 0.05\, N$$

Give a brief interpretation of the obtained coefficients.

$a = k$ - coefficient of elasticity of the spring: $k = 23 \pm 1 \frac{N}{m}$

b - coefficient that arises due to the fact that the spring has an initial length

6.3. Write down in the table the coordinates (x_1 and x_2) of the markers on the spring and the readings of the multimeter at each stage of stretching the wire (U_{AB}' - the amplified voltage between points A and B).

x_1, cm	x_2, cm	U_{AB}', mV	x, cm
17.3	25.2	196	7.9
16.4	25.2	205	8.8
15.4	25.1	216	9.7
14.3	25.1	226	10.8
13.4	25.1	237	11.7
12.5	25.0	247	12.5
11.4	25.0	260	13.6
10.5	25.0	270	14.5
9.5	25.0	279	15.5
8.4	25.0	295	16.6
7.4	24.9	309	17.5
6.4	24.9	321	18.5
5.5	24.9	338	19.4
4.5	24.9	352	20.4
3.5	24.8	379	21.3
2.5	24.7	579	22.2
1.5	24.4	886	22.9
0.5	22.8	1160	22.3
-0.5	22.2	1466	22.7

Calculate the length of the spring (x) at each stage of deformation and also enter this data in the same table.

Calculate and write down the cross-sectional area (S) of the wire, which corresponds to the diameter of the wire obtained in part 3 of the work.

$$S = \pi\frac{d^2}{4} = (7.4 \pm 0.7) \cdot 10^{-9}\ \mathrm{m}^2$$

Write down the voltage (U_0) measured on the unstretched wire before the experiment.

$$U_0 = 1277\ \mathrm{mV}$$

6.4. Derive the formula expressing the dependence of the amplified voltage between points A and B (U_{AB}') in terms of the following values:

- the current length of the spring (x),
- the initial length of the spring (x_0),
- coefficient of elasticity of the spring (k),
- amplifier gain (G),
- power supply voltage (U),
- initial voltage on unstretched wire (U_0),
- Young's modulus of the wire (E) and
- the cross-sectional area of the wire (S).

Also use the dependence of the resistance of the wire section on its length obtained in part 4 of the work. Remember that you should get the dependence of the change in the resistance of the wire in the branch of the bridge circuit on the voltage between points A and B in the previous part of the work.

Write down the derivation of the formula:

The dependence of the resistance of the wire section on its length obtained in part 4 of the work: $\ln R = k' \ln L - const$, $k' = 1.90$

$$\frac{dR}{R_0} = k' \cdot \frac{dL}{L}$$

$$dU_{AB} = \frac{U_0(U - U_0)}{U} \cdot \frac{dR}{R_0} \rightarrow \frac{dR}{R_0} = \frac{U_{AB} U}{U_0(U - U_0)}$$

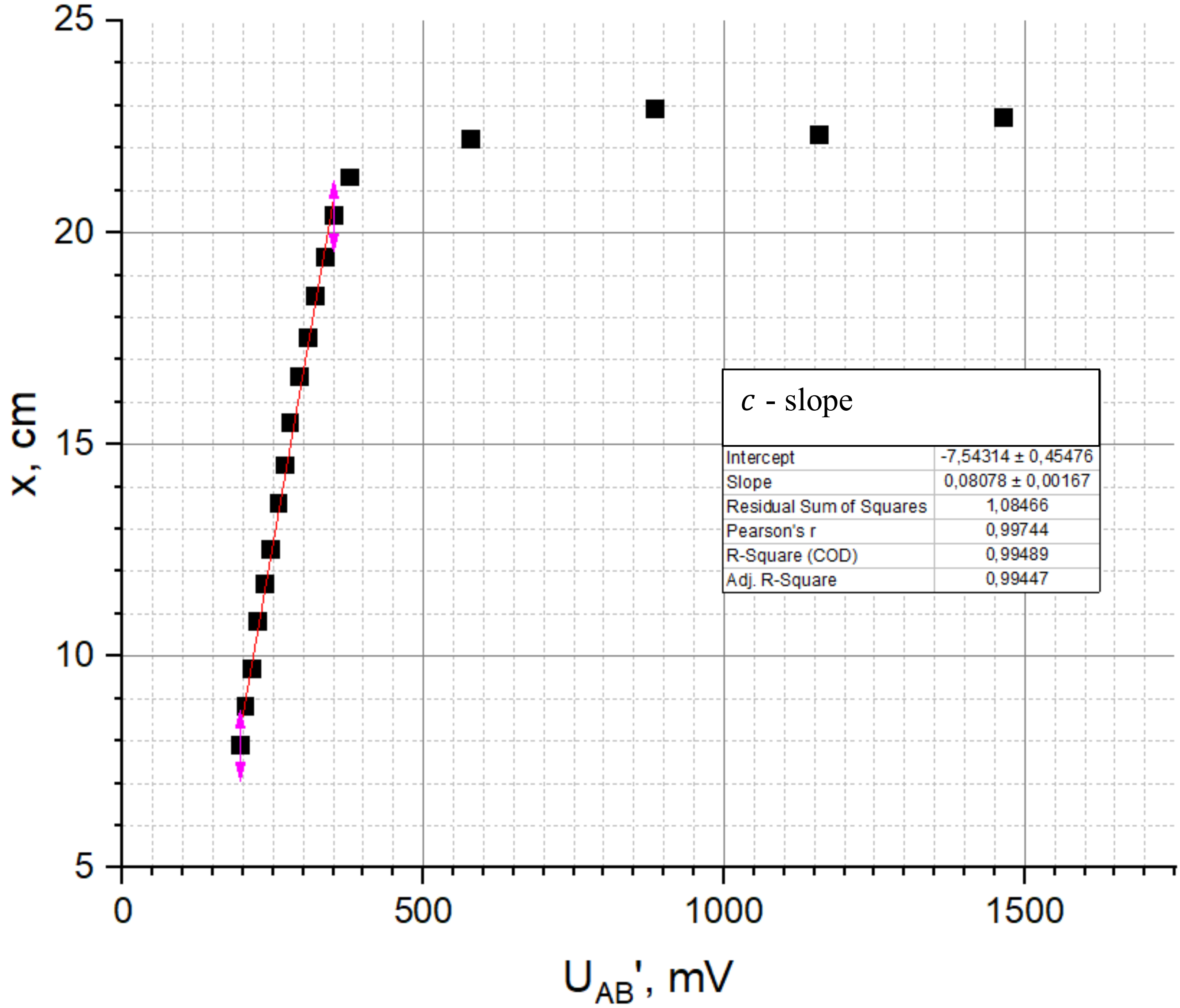
$$\frac{U_{AB} U}{U_0(U - U_0)} = k' \cdot \frac{dL}{L} = k' \cdot \frac{\Delta F/S}{E}$$

$$U_{AB}' = G \cdot U_{AB} = G \cdot k' \cdot \frac{\Delta F/S}{E} \cdot \frac{U_0(U - U_0)}{U}$$

$$\Delta F = k \cdot x$$

$$U_{AB}' = G \cdot U_{AB} = G \cdot k' \cdot \frac{kx/S}{E} \cdot \frac{U_0(U - U_0)}{U}$$

6.5. Plot the dependence of the length of the spring on the amplified voltage between points A and B ($x(U_{AB}{}')$).



Write down the slope of the resulting graph (c):

$$c = 0.081\ \frac{cm}{mV} = (0.81 \pm 0.08)\ \frac{m}{V}$$

6.6. Express the Young's modulus (E) from the formula obtained in paragraph 6.4 through the slope (c) of the plotted graph.

$$U_{AB}' = G \cdot k' \cdot \frac{kx/S}{E} \cdot \frac{U_0(U - U_0)}{U}$$

$$\frac{1}{c} = G \cdot k' \cdot \frac{k/S}{E} \cdot \frac{U_0(U - U_0)}{U}$$

$$E = \frac{G \cdot k' \cdot k \cdot c}{S} \cdot \frac{U_0(U - U_0)}{U}$$

Calculate the value of Young's modulus (E).

$$E = \frac{40 \cdot 1.90 \cdot 23.1 \cdot 0.81 \cdot 1.277 \cdot (2.5 - 1.277)}{2.5 \cdot 7.41 \cdot 10^{-9}} = (1.2 \pm 0.1) \cdot 10^{11}\ \frac{N}{m^2}$$

6.7. Using the graph, determine the maximum wire tension (F_{max}) at which its deformations can still be considered linearly dependent on the applied force.

$$F_{max} = x_{max} \cdot k + b = 0.21 \cdot 23.1 - 1.06 = 3.8 \pm 0.3\ \text{N}$$