

OLYMPIAD OF METROPOLISES
Moscow, September 4

1. Unsaturated acid A participates in the Tricarboxylic acid cycle (the Krebs cycle). Maleic acid is a geometric isomer of A. How many grams of malic acid can be obtained by hydration of 58 g of A? Give the integer answer without units.
2. Which sort algorithm has a worst-case time behavior for exchanges $O(N)$?
 1. Bubble sort
 2. Merge sort
 3. Selection sort
 4. Quick sort
 5. There is no such sort algorithm

3. Curious physics students, who like mathematics, took 12 identical capacitors with capacitance C each and connected them in the shape of octahedron. They were wondering what the equivalent capacitance of such a network would be if measured between two adjacent nodes. Help them by giving a numerical answer in units of C accurate to the first decimal.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

4. How many "1" digits are in the binary representation of the following hexadecimal expression $FACE - BED + BAD$?
5. The diagonals AC and CE of a regular hexagon $ABCDEF$ are divided by inner points M and N respectively, so that $\frac{AM}{AC} = \frac{CN}{CE} = r$. What is the value of $21r^2$ if B , M , and N are collinear?
6. An iron plate with the mass of 5.00 g was placed into the solution containing 3.25 g of iron (III) chloride and 4.80 g of copper sulfate. Once the reactions have gone to completion, the plate was picked out of the solution, dried and weighed. What is its mass?
 1. 2.76 g
 2. 4.68 g
 3. 5.00 g
 4. 5.24 g

7. An electrical device is designed to be connected to AC network with a voltage of 127 V and to release a power of 100 W. What power will be released by the device if it is connected in series with an ideal diode to AC network with a voltage of 220 V?

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8. How many asymmetric carbon atoms (chiral carbons, chirality centers) are found in 1,3-dichloro-4-methyl-2-pentanol?
Give the integer number.

9. Provide the output of the following program:

```
include <stdio.h>
int main() {
    int val[10] = 0, 0, 0, 0, 0, 0, 0, 0, 0, 0;
    int sum = 0;
    int i;
    for( val[4] = 0; val[4] < 10; val[4]++) {
        val[val[4]] = val[4] * 2;
    }
    for( i = 0; i < 10; i++) {
        sum = sum + val[i];
    }
    printf( %d, sum);
}
```

10. For a triangle ABC with incenter I , denote by D the midpoint of the minor arc BC on the circumcircle. Let $\angle BIC = 120^\circ$, $ID = 7/\sqrt{3}$ and $BC = AB + 2$. Find the value of $AI \cdot \sqrt{3}$.
11. In The Nine Chapters on the Mathematical Art, a Chinese mathematics book composed by several generations of scholars from the 10th - 2nd century BCE, an algorithm called "Subtracting Each Other Technique" was recorded. Here is an implementation of this algorithm in Visual Basic.

```
Private Sub Command1_Click()
    Dim a,b,c,r,t As Integer
    a = Val(TextBox("Input The First Number"))
    b = Val(TextBox("Input The Second Number"))
    c = 0
    Do While a Mod 2 = 0 And b mod 2 = 0
        a = a / 2
        b = b / 2
        c = c + 1
    Loop
    Do While a <> b
        If a < b Then
            t = a
            a = b
            b = t
        End If
        r = a - b
        a = b
        b = r
    Loop
    Print r*2^c
End Sub
```

Provide the output of the program after entering two numbers,
 $a = 98$ and $b = 63$.

12. Sherbet lemons are sweets which consist of a flavored sugar shell filled with sherbet. Sherbet contains sodium hydrogen carbonate and tartaric (2,3-dihydroxybutanedioic) acid. Sherbet produces a slight fizzing sensation in the mouth when the tartaric acid reacts with the sodium hydrogen carbonate to give carbon dioxide. In a laboratory experiment, one sherbet lemon sweet produced 6.00 cm^3 of carbon dioxide ($25 \text{ }^\circ\text{C}$, 99 kPa).

Calculate the minimum mass of the mixture of tartaric acid and sodium hydrogen carbonate necessary to produce this volume of carbon dioxide.

- 38.1 mg
 - 42.6 mg
 - 56.1 mg
 - 62.7 mg
13. The athlete was photographed from a distance of 20 m by a camera, with a focal length of 45 mm and the exposure of 0.02 s. The image on the film was blurred and the blur turned out to be 1 mm. At what speed was the athlete moving perpendicular to the main optical axis of the lens?

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14. A figure in the coordinate plane is given by

$$\left|y - \frac{1}{2}x^2\right| + \left|y + \frac{1}{2}x^2\right| \leq 2 + x.$$

Find the area of the figure.

15. The electric potential at a point X some distance away from a point charge C is 2 V. The distance between points Y and C is a quarter of the distance between X and C . Determine the work done when a test charge equivalent to that of 3 protons moves from the point X to the point Y . Express the answer in electronvolts.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

16. A trolley of a mass of 400 kg begins to move from the state of rest. During its motion different forces are acting on it, so that the net force uniformly increases from 0 to 80 N in 10 seconds. Determine the trolley speed 10 seconds after it started to move.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

17. At the extensions of the sides AB , BC , CD , DA of a convex quadrilateral $ABCD$ beyond the points B , C , D , A respectively points M , N , P , Q are marked such that $BM = AB$, $CN = 2BC$, $DP = CD$, $AQ = 2AD$. Find the area of the quadrilateral $MNPQ$ given that the area of $ABCD$ is 4.

18. You have a fifteen-story building and three identical hard drives. And you have a task of finding the highest floor one from which you can drop a hard drive without losing its functionality. If it breaks when dropped from the first floor, then the answer is 0. You can drop hard drives you have from whichever floor you want and then you can check if they are still working. What is the least number of drops required to find the wanted floor?

All hard drives break from the same height and this height does not change while the hard drive is still operational. This means that, if you drop a hard drive and it is still operational you can drop it from a different floor.

You can start dropping hard drives from the first floor, then the second, then the third and so on, until it breaks. This way you can find the answer in 15 drops. Of course, you can do it in fewer drops.

19. An aldehyde $C_nH_{2n}O$ reacted with an excess of silver oxide ammonia solution. Acidification of the resultant solution led to liberation of a gas. Find n .

20. Determine x^2 , if x satisfies the equation

$$\arcsin(1/3) + 2 \arctan(1/\sqrt{2}) = \arccos(x) + \arccos(2x).$$

21. Natural chlorine contains two isotopes: chlorine-35 with the atomic mass of 34.97 Da and chlorine-37. The abundance of the former isotope is 75.78 at.%. The average atomic mass of chlorine is 35.45 Da. What is the atomic mass of chlorine-37?

1. 36.85 Da
2. 36.95 Da
3. 37.00 Da
4. 37.06 Da

22. There are 20 lanes, numbered 1 to 20, and 4 runners called A, B, C, and D. The race organizers call a lane assignment *optimal* if there are at least 3 empty lanes between any two runners; i.e., whenever lanes i and j are assigned to some two runners, we have $|i-j| \geq 4$. (For example, the assignment of lanes 15, 1, 5, 20 to the runners A, B, C, D, respectively, is optimal.) How many optimal lane assignments are there?

23. When two resistors are connected in parallel their total resistance is 6.25 times less than that of their connection in series. What is the ratio of the resistor values?

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24. 32-bit CPU has a 32-bit address bus. What is the maximum amount of memory it can address (in bytes)?

25. How many integer solutions does the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{19}$ have?

26. Find $f(1)$, given $f(x) = f(x+2) + x$ if $x < 6$
and $f(x) = x - 1$, otherwise.

27. What is the minimum number of bits needed to represent a set of 20000 different items?

28. Which of the following compounds give alkaline solutions?

1. sodium sulfate
2. sodium sulfide
3. sodium carbonate
4. sodium hydrogen carbonate
5. sodium hydrogen sulfate

29. How many moles of propanoyl-chloride have reacted with an excess of ethanol if the product contains $1.2 * 10^{23}$ oxygen atoms?

1. 0.05 mol
2. 0.1 mol
3. 0.2 mol
4. 0.4 mol

30. Array A has 6 numbers indicating the daily maximum temperatures in Shanghai in a period of time: $a[1] a[2] a[3] a[4] a[5] a[6] = \{35, 33, 34, 36, 35, 32\}$. Array A is going to be sorted in descending order, and bubble sort algorithm that compares two adjacent numbers from right to left is going used. What is the value of the fourth element in Array A after the second round of data processing.

During the k -th round of data procession, assuming there are n elements, first we compare the $(n-1)$ -th and n -th element and swap them if necessary, then $(n-2)$ -th and $(n-1)$ -th, etc. and at the end of the k -th round we compare and swap the k -th and the $(k+1)$ -th element.

31. Find the two's complement of 2017 (a binary signed number representation of -2017) if it is 16-bit integer.
32. Some electric current is flowing in a 1 m long wire. The wire is placed perpendicular to a magnetic field of 0.2 T. If the wire experiences a force of 0.6 N, what is the magnitude of the current flowing in the wire?
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33. Choose the process(es) decreasing the system entropy.
1. Evaporation of an alcohol
 2. Dissolution of hydrogen chloride in water
 3. Condensation of water vapor
 4. Iron melting
 5. Ammonia synthesis from elements
34. 1 mole of a non-cyclic hydrocarbon C_nH_m contains $(8n-4)$ moles of electrons. How many π -bonds are found in the hydrocarbon molecule?
- Give the integer number.
35. A red ball moving at the velocity of 2 m/s collides head on with a yellow ball of the same mass standing still. What is the velocity of the yellow ball after the collision if the red ball stops?
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 $g = 10 \text{ m/s}^2$.
36. Several people attended a high school reunion. All attendees shook hands with each other. The total number of handshakes was 435. How many people attended the reunion?
37. A is the positive integer such that $\text{LCM}(100, A) = 600$ and $\text{LCM}(100, A + 1) = 100$.
 Find the value of $\text{LCM}(100, A + 2)$.
38. How many four-digit multiples of 9 consisting of four different odd digits are there?
39. A chemical element contains equal number of filled and half-filled orbitals in its ground state. Find any element with this property. Give the answer as the element number in the periodic table.
40. A metal ring of a radius of 0.1 m is lying still on a table. The electrical resistance of the ring is 1.57 Ohm. Find the electric charge (in microcoulombs) which would flow in the ring if we quickly turn it around, so that it ends up lying on its opposite side. The vertical component of the Earth's magnetic field is $5 \cdot 10^{-5} \text{ T}$.
- The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.
41. The negabinary representation of a number n is its representation in base -2 using digits 0 and 1 only. For example, the negabinary representation of number 7 is 11011. It is known that every integer has exactly one negabinary representation. You are given negabinary representations of two numbers: 1011 and 1101. Find the negabinary representation of their product.

42. Five gases are given below:

1. N_2
2. H_2
3. O_2
4. CH_4
5. NH_3

Choose those 1 gram of which is heavier than 1 liter (0°C , 1 atm)?

43. Consider the following sequence:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, \dots$$

What is the 100th term?

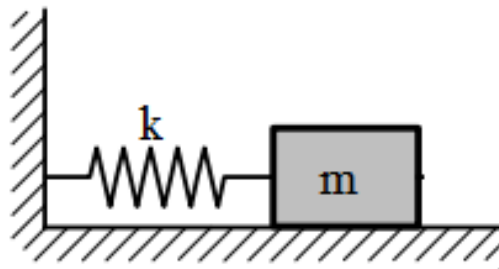
44. A boat of a length of 5 m and a mass of 120 kg is at rest in still water. An 80 kg sailor is walking in the boat from one end to the other. Find the distance in meters which the boat would move relative to the shore when the sailor reaches the boat end. The friction between the boat and water is negligible.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
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45. A tape is 8 cells long and 1 cell wide. Cells of the tape contain numbers 4, 9, 20, 17, -9, 9, 20, 17 in this order. Two players play a game. By turns each player cuts off either leftmost or rightmost cell and takes the cut cell. At the end of the game the number of points got by the player is the sum of numbers on all taken cells. How many points can the first player guarantee himself regardless of the opponent strategy?

46. Find the sum of quantum numbers n, l, m_l for all the valence electrons of As atom in its ground state.

47. A spring pendulum consisting of a block of a mass $m = 200 \text{ g}$ and a spring with spring constant $k = 30 \text{ N/m}$ lies on a horizontal surface (see Fig.). The coefficient of friction between the block and the table is $\mu = 0.12$. At the initial moment the spring is not deformed. Then the block is displaced from the initial position at a distance $A = 5.5 \text{ cm}$ and released.



Determine the distance travelled by the block before it stops. Give the answer in centimeters.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

48. On a ring road there are four gas stations: A, B, C and D. The distance between A and B is 75 km, between A and C — 50 km, between C and D — 40 km, between D and A - 60 km. Find the distance between B and C. (The distances are measured in the shorter direction along the road.)

49. Four solutions are given below:

1. 0.1 M acetic acid ($pK_a = 4.75$)
2. 0.1 M formic acid ($pK_a = 3.75$)
3. 1 M formic acid ($pK_a = 3.75$)
4. 0.00066 M ammonia ($pK_b = 4.75$)

Choose the solution with the highest degree of ionization and calculate its pH. In the answer, give only pH rounded to the integer value.

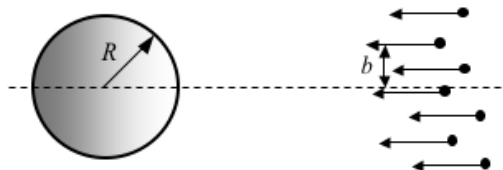
50. A physicist, chemist, biologist and a mathematician are out walking. They come to an old bridge. The bridge is weak and can carry only two of them at the same time. Since they are late to the university and the light is fading they must cross in the minimum time possible and must carry a flashlight on every crossing. They only have one flashlight and it can only be delivered by hand (e.g. after the first two cross, one has to return with the flashlight). Because of their different age, each one crosses the bridge in a different speed. The physicist can cross in 1 minute, the chemist in 2 minutes, the biologist in 5 minutes and the mathematician in 10 minutes.

What is the minimum possible time for all of them to cross the bridge, if they perform optimally?

51. A parallel monochromatic electron beam strikes a metal sphere of radius $R = 10$ cm. Assume that the sphere charge equals half the maximum charge to which the sphere is eventually charged by the given beam.

Determine the maximum distance b from the axis passing through the sphere centre and parallel to the incident beam at which the electrons still reach the sphere. (The distance b is measured far away from the sphere where electron velocities are parallel.) Give the answer in centimeters.

If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.



52. In an arithmetic sequence, the arithmetic mean of the first ten terms equals 6, while the arithmetic mean of the first twenty terms equals 16. Find the 15th term of the sequence.

53. Mercury and water of the same mass have been poured into an open cylindrical vessel. The total height of the two layers of the liquids is 29.2 cm. Determine the pressure (in Pa) on the bottom of the vessel. Density of mercury is 13.6 g/cm^3 ; water density is 1.0 g/cm^3 .

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

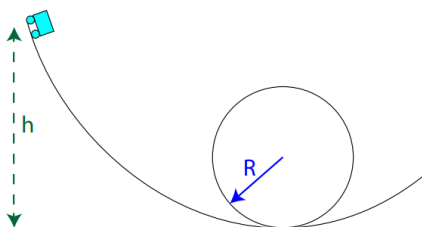
54. In this problem, we want to do the X^N operation with the least number of multiplications. We can use only the value of X and the values we calculated before.

Example for $N = 5$:

- 1) $X \times X = X^2$
- 2) $X^2 \times X = X^3$
- 3) $X^3 \times X^2 = X^5$

Write the minimal number of multiplications for $N = 27$.

55. A roller coaster car starts at rest at height h above the ground and completes a loop along its path. Determine the minimum value of h (in the units of loop radius R), in order for the car to remain on the track throughout the loop. Neglect friction.



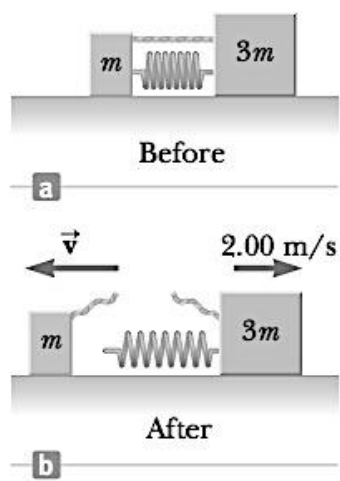
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 $g = 10 \text{ m/s}^2$.

56. Look through the following algorithm:

1. INPUT n
2. PRINT n
3. IF $n = 1$ THEN STOP
4. IF n is odd THEN $n = 3n + 1$
5. ELSE $n = n/2$
6. GOTO 2

How many numbers will be outputted if input is 22?

57. Two vessels of the same volume are kept at the same temperature. Vessel A contains CH_4 , whereas vessel B — O_2 . Masses of the gases are equal. Choose only one true statement.
1. Both vessels contain an equal number of molecules
 2. The pressures in the vessels are equal
 3. The pressure in the vessel B is twice higher than that in the vessel A
 4. The pressure in the vessel A is twice higher than that in the vessel B
58. Two blocks of masses m and $3m$ bound by a massless string with a spring being compressed between them are placed on a frictionless horizontal surface as shown in the figure (a) below. When the string is cut as shown in figure (b), what will the speed of the mass m be?



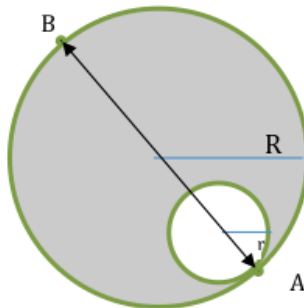
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59. If x and y are positive, find the minimal value of the function

$$f(x, y) = \frac{16x^3}{y} + \frac{y^3}{x} - 8\sqrt{xy} + 6.$$

60. It is given that 67 in base- x is equal to 51 in base- y , what are the smallest x and y for which the equality is verified? Give the answer as a single number concatenating x and y (e.g. if $x=10$ and $y=8$ write the answer as 108).

61. A planet of radius R contains a spherical cavity of radius r which shares a common tangent plane with the planet surface at point A . An astronaut throws a rock at point A and finds out that $g(A) = 25\text{m/s}^2$, where $g(X)$ is the gravitational acceleration at point X . Given that $r = R/3$ find $g(B)$ (the gravitational acceleration at point B).



The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

62. Which of the following sorting algorithms achieve(s) a worst-case time complexity of $O(N \log N)$, when sorting N numbers in ascending order?

1. Bubble sort
2. Merge sort
3. Selection sort
4. Quick sort
5. Heap sort

63. Choose one of the hereunder particles with the sp^3d hybridized central atom.

1. XeF_4
2. SO_4^{2-}
3. SF_4
4. SiF_6^{2-}

64. Evaluate the definite integral

$$\int_0^1 x^{2x} (\ln(x) + 1) dx.$$

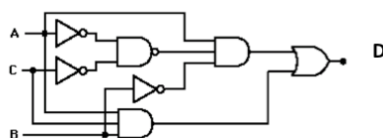
(Here $\ln(x)$ denotes the natural logarithm.)

65. There are 24 points given in space, no three of which are collinear. We mark every plane containing at least three of the given points. Given that not more than 2017 planes are marked, determine the largest possible number of marked planes.

66. Determine which crystal has a higher index of refraction if angles of incidence and angles of refraction of a laser beam that goes through three crystals are:

- 1) 50° and 30° ,
- 2) 30° and 18° ,
- 3) 60° and 38°

67. The following circuit board receives A, B and C inputs. Logic gates produce D value:



First, complete the Truth table based on A, B and C values (0=false, 1=true).

What is the sum of elements in the Truth table D column?

NOT gate	A		out
AND gate	A		out
OR gate	A		out
NOT AND gate	A		out

For reference:

68. Convert octal number 1147 from base 8 to base 16.

69. Turan and Samir exercised on the same route. Samir jogged the first half of the route at 6 km/h and ran the other half at 12 km/h, taking a total time of x . Turan walked the first third of the route at 5 km/h and ran the rest of the route at 15 km/h, taking a total time of y . Find $x : y$.

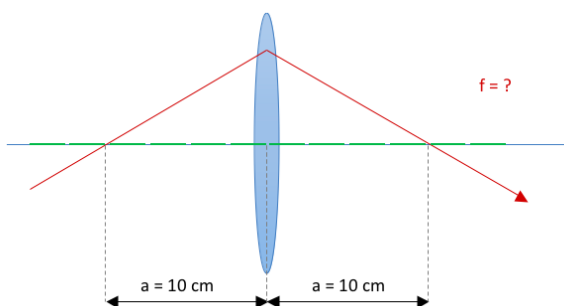
70. A friction force of 1 N acts for 2 s on a 500 g hockey puck which initial speed is 10 m/s. Determine the distance traveled by the puck during this period.

The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal.
 $g = 10 \text{ m/s}^2$.

71. Which of the statements are true for the hereunder octahedral complexes?

1. There are two geometric isomers for $[Fe(en)_2Cl_2]^+$ ($en = NH_2CH_2CH_2NH_2$ is a bidentate ligand)
2. The oxidation state of iron is +2 in $[Fe(en)_2Cl_2]^+$
3. $[Fe(en)_2Cl_2]^+$ is diamagnetic, because en is a strong field ligand
4. There are 6 isomers for $[Pt(CN)_2(H_2O)_2Br_2]$

72. A thin converging lens refracts the ray of light symmetrically as shown in the figure. Calculate the focal length of the lens (in cm).



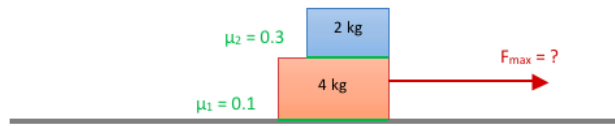
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 $g = 10 \text{ m/s}^2$.

73. For how many integer a the equation $2x^3 + x^2 - 20x + a = 0$ has exactly 3 different real solutions?

74. The zinc-copper alloy has a face-centered cubic lattice. In the unit cell, all the vertices are occupied by zinc, and all face centers by copper atoms. What is the Cu/Zn atomic ratio in the alloy? Give the integer value.

75. If $\cos(x) = 0.1$ and $0 \leq x \leq \pi/2$, then $\sin(x) = 0.3\sqrt{t}$. Determine t .

76. A brick of a mass of 2.0 kg lies on top of a bigger one of 4.0 kg. The coefficient of static friction between the horizontal surface and the bigger brick is $\mu_1 = 0.10$ and the coefficient of static friction between the two bricks is $\mu_2 = 0.30$. Figure out the maximum force to pull the pile of bricks without shifting the upper one. The force is applied at the point of no torque, i.e. where all force moments add to zero.



The answer should be given in the SI units unless otherwise stated. If the numerical result turns out to be an integer, it should be given as the answer. Otherwise round the result to the first decimal. $g = 10 \text{ m/s}^2$.

77. An ionic compound is composed of three elements and contains 61.3 mass.% of fluorine. Determine the compound formula. Give the answer as its molar mass rounded to the nearest integer, without units.
78. Each edge of a regular square pyramid $SABCD$ (with apex S) is of length 5. Points P, Q, R are taken on the edges SA, AB, BC respectively such that $PA = AQ = RC = 2$. Find the distance from the vertex D to the plane PQR .
79. Thermal decomposition of $Ba(IO_3)_2$ leads to $Ba_5(IO_6)_2$ and two elements. Decide on the reaction equation. Give the answer as the sum of all the coefficients being sure that these are the minimum possible integers.
80. Giray had three 100-lira banknotes and two 50-lira banknotes in his right pocket. In his left pocket, he had four 50-lira banknotes and two 100-lira banknotes. Giray moved a random banknote from the left pocket to the right one, and then moved a random banknote from the right pocket to the left one. After that a random banknote was stolen from his right pocket. How many times is the probability that a 100-lira banknote was stolen greater than the probability that a 50-lira banknote was stolen?