2nd Olympiad of Metropolises

Mathematics · Day 1

Problem 1. Let $ABCD$ be a parallelogram in which the angle at B is obtuse and $AD > AB$. Points K and L are chosen on the diagonal AC such that $\angle ABK = \angle ADL$ (the points A, K, L, C are all different, with K between A and L). The line BK intersects the circumcircle ω of triangle ABC at points B and E, and the line EL intersects ω at points E and F. Prove that $BF \parallel AC$.

Problem 2. In a country there are two-way non-stop flights between some pairs of cities. Any city can be reached from any other by a sequence of at most 100 flights. Moreover, any city can be reached from any other by a sequence of an even number of flights. What is the smallest positive integer d for which one can always claim that any city can be reached from any other one by a sequence of an even number of flights not exceeding d?

(It is allowed to visit some cities or take some flights more than once.)

Problem 3. Let $Q(t)$ be a quadratic polynomial with two distinct real zeros. Prove that there exists a non-constant polynomial $P(x)$ with the leading coefficient 1 such that the absolute values of all coefficients of the polynomial $Q(P(x))$ other than the leading one are less than 0.001.