The 4th Olympiad of Metropolises
Mathematics

Problems

Day 1

Problem 1. Three prime numbers p, q, r and a positive integer n are given such that the numbers

$$\frac{p+n}{qr}, \quad \frac{q+n}{rp}, \quad \frac{r+n}{pq}$$

are integers. Prove that $p = q = r$. *(Nazar Agakhanov)*

Problem 2. In a social network with a fixed finite set of users, each user has a fixed set of *followers* among the other users. Each user has an initial positive integer rating (not necessarily the same for all users). Every midnight the rating of every user increases by the sum of the ratings that his followers had just before the midnight.

Let m be a positive integer. A hacker, who is not a user of the network, wants all the users to have ratings divisible by m. Every day, he can either choose a user and increase his rating by 1, or do nothing. Prove that the hacker can achieve his goal after some number of days. *(Vladislav Novikov)*

Problem 3. In a non-equilateral triangle ABC point I is the incenter and point O is the circumcenter. A line s through I is perpendicular to IO. Line ℓ symmetric to the line BC with respect to s meets the segments AB and AC at points K and L, respectively (K and L are different from A). Prove that the circumcenter of triangle AKL lies on the line IO. *(Dušan Djukić)*