Problem 1: CARBONATED WATER

Suppose you have a regular 1-liter factory sealed bottle of carbonated water. You have turned the bottle cap slightly to unscrew it (so a hissing sound was heard) and screw the cap tightly again. Now you would observe bubbles of carbon dioxide (CO_2) rising upwards – large, at first, and then smaller ones. Let us study the process of bubble surfacing.

One can easily see that the shape of a small bubble is much closer to spherical than that of a bigger one.

1) Estimate the maximal size of an immobile bubble such that the bubble shape approximates sphere with an accuracy of 10% or better. Water density is $\rho \approx 1$ g/cm^3, the surface tension $\sigma \approx 0{,}07$ N/m, and the free fall acceleration is taken to be $g \approx 9{,}8$ m/s^2. The numerical answer should be in mm.

Consider a bubble so small that it can be regarded as almost spherical. For instance, let the bubble initial diameter near the bottle bottom be $d_0 = 0{,}3\,\text{mm}$.

2) Figure out the bubble acceleration right after it has detached from the bottom. The CO_2 density inside the bubble at the given water temperature is $\rho_v \approx 0{,}002$ g/cm^3. The numerical answer should be in m/s^2.

Drag force exerted on a bubble, when it is moving in water, is linearly proportional to its cross-sectional area, water density, and the bubble velocity squared. We assume the proportionality factor for this problem to be $\beta = 0{,}2$.

3) Suppose that the bubble volume remains constant. What is the terminal velocity the bubble can reach? Write down the equation and evaluate the numerical value (in m/s).
4) Estimate the time it takes the bubble to reach the terminal velocity after detachment, i.e. when its acceleration becomes much less than g. Write down the equation and evaluate the numerical value (in seconds).

Actually, the density of CO_2 molecules dissolved in a liquid is much higher than that in a gas bubble. Therefore, the dissolved gas diffuses into the bubble and its radius grows. It is reasonable to assume that the growth rate of the bubble volume due to the diffusion is proportional to the bubble surface area and to an excess of the dissolved gas density and inversely proportional to the thickness of the liquid layer through which the gas diffuses (called the «depletion layer»). The faster bubble is moving the thinner is the effective depletion layer (due to circulation of water surrounding the bubble). A kinetic theory calculation gives for the depletion layer thickness: $\delta = const \cdot \sqrt{\frac{d}{v}}$, where v is the bubble velocity and d is its diameter.

5) Suppose it takes time T for a bubble to surface. Figure out the bubble diameter as a function of time t, providing it has increased by a factor $k = 3^{4/5} \approx 2{,}4$ during the surfacing. The answer should be an equation for $d(t)$ in terms of T, t, d_0, and $k = 3^{4/5}$. Neglect a change in the density of CO_2 dissolved in water during the bubble surfacing.

6) Determine time dependence $v(t)$ of the bubble velocity (the answer should include a formula for $v(t)$ expressed in terms of the parameters listed above and the terminal velocity v_0).
7) Determine the law of motion of a bubble, i.e. time dependence of its elevation h above the bottom (the answer should include a formula for $h(t)$ in terms of T, t, and v_0; take the same value $k = 3^{4/5}$).
8) Suppose the height of water column, which the bubble traverses on its way upward, equals $H = 30$ cm. Evaluate the time of bubble surfacing (the answer should be the explicit formula including the parameters given in the problem and calculate the numerical value in seconds).

During bubble surfacing *some heat is being released* (the drag force does a work by increasing the kinetic energy of turbulent flow which eventually dissipates as heat) and at the same time *some heat is being absorbed* (due to CO_2 evaporation from water into a bubble).

9) Assume that all the heat *released* during the surfacing is converted into heating the «column» of water which cross-section equals the average cross-section of a rising bubble. Using this assumption estimate (by the order of magnitude) the temperature increment of water in the «column». The specific heat capacity of water is $c_w \approx 4200$ J/kg·K. The answer should be given in Kelvin.
10) Evaporation into bubbles of 1 mole of CO_2 dissolved in water requires approximately 20 kJ of energy. Estimate (by the order of magnitude) the cooling of the water «column» (see 9)) due to this effect during the bubble rising and compare to the heating effect (see 9)). The answer should be given in Kelvin. What will the net result be? The answer should be either «+» (the temperature increases) or «–» (the temperature decreases). Assume that the pressure and temperature in the bottle change slightly and remain close to $p_0 \approx 120$ kPa and $T_0 \approx 290$ K.

Problem 2: RADIOGRAPHY

In 1943 there was founded «Laboratory № 2» of the USSR Academy of Sciences, a Soviet physicist Igor Vasilyevich Kurchatov was appointed as its director. The laboratory mostly concentrated on the development of nuclear reactor and nuclear weapons. Since then many years have passed, the laboratory turned into a large research center and its name has changed several times. Now it is called the National Research Centre "Kurchatov Institute", one of the largest Russian scientific centers which does research in a variety of fields. The main area of the institute research includes solid-state physics and materials science.

Industrial radiography is one of the basic modern methods of materials science. A studied object is placed in a coherent monochromatic beam of X-ray photons of high intensity which scattering pattern is then analyzed. Sometimes *methods of spectrometry* are used, i.e. variation of intensity of the beam passing through a material is measured as a function of radiation wavelength. However, the most common methods of study of atomic structure are *diffraction methods*. They are based on analysis of the diffraction pattern resulting from elastic scattering of X-rays by atoms of the sample. Notice that radiation wavelength remains constant in elastic scattering. A common source of coherent X-rays of high intensity with a wide wavelength spectrum is *synchrotron* – a large ring storage of charged particles traveling at a speed close to the speed of light. Such particles are called *relativistic* because their motion is no longer described by Newtonian laws of classical mechanics, instead one must use the special theory of relativity (STR). The operating principle of synchrotron is based on the fact that a charged particle emits electromagnetic radiation at trajectory turns. Direction of particle velocity is changed by special bending magnets. A particle (e.g. electron) path in the synchrotron ring consists of straight segments, where electrons receive kinetic energy, and segments of almost constant curvature in a strong magnetic field of bending magnets. In the curved segments electron motion is highly accelerated, so they emit electromagnetic radiation in the X-ray range. A powerful synchrotron is operating at the Kurchatov Institute in Moscow.

Relativistic equation of motion of a charged particle in magnetic field is $\frac{d\vec{p}}{dt} = \overrightarrow{F_L} = q[\vec{v} \times \vec{B}]$, where the particle momentum $\vec{p} = \frac{m_0\vec{v}}{\sqrt{1-v^2/c^2}} \equiv m\vec{v}$. Here c is the speed of light in vacuum ($c \approx 3 \cdot 10^8 m/s$), the quantity m_0 is the particle *invariant mass* and $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ is the particle *relativistic mass*. The energy of relativistic particle $E = \frac{m_0c^2}{\sqrt{1-v^2/c^2}} = mc^2$. Usually the energy of a microparticle is measured in electronvolts: 1 eV ≈ $(1{,}6\cdot10^{-19}$ C)·(1 V). The factor $\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$ is called the «*Lorentz factor*». If the particle speed is close to c, $\gamma \gg 1$.

1) Evaluate the Lorentz factor for an electron of energy E_e = 2,5 GeV (the invariant mass is $m_0 \approx 9 \cdot 10^{-31}$ kg, *the rest energy* is $m_0c^2 \approx 0{,}5$ MeV). By how many percent is the speed of such an electron less than c? The electron charge is $e \approx 1{,}6 \cdot 10^{-19}$ C. The answer should include formulae and numerical values.

2) Determine the curvature radius of electron trajectory in the field of bending magnet if electron energy in the synchrotron storage ring is maintained at E_e = 2,5 GeV and the induction of the field of bending magnet is B = 1,7 T. Electrons are traveling in a plane perpendicular to the magnetic field lines. The answer should include the formula and the numerical value (in meters).

Any accelerating charged particle emits electromagnetic radiation. The important feature of synchrotron radiation (i.e. radiation of relativistic particles with $\gamma \gg 1$ traveling along a curved path) is its «searchlight» nature: almost all the energy is radiated «forward» along the particle velocity in a narrow cone with half an aperture of $\varphi \approx \frac{1}{\gamma}$ (see Fig.1).

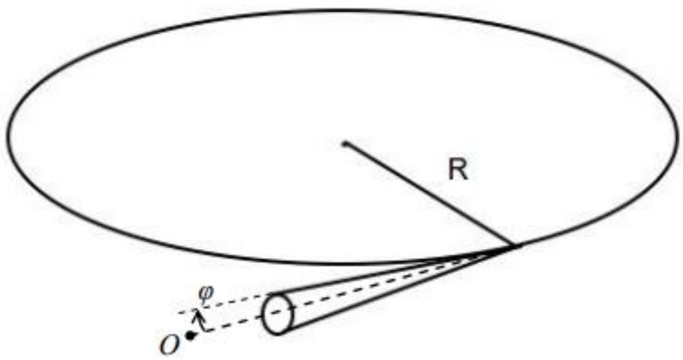


Fig. 1.

3) Suppose an «observer» O resides at the circular orbit plane and can be regarded as a point. In this case he (or she) detects radiation flashes corresponding to the short periods when she is inside the «searchlight» cone of orbiting particle. Determine the length Δl of the arc traversed by

electron on the orbit when its radiation is detected by the observer. The electron energy and the magnetic induction are given in 2). The answer should be the equation.

4) Determine duration T_{sr} of the radiation «flash» detected by the observer. It must be taken into account that due to electron relativistic motion it takes different time for the «initial» and «final» portions of the «flash» to reach the observer. The answer should include the equation and the numerical value (in seconds).

Thus, radiation of relativistic particle traveling along a circular path is observed as a bright short flash. The flash spectrum (frequencies and wavelengths) turns out to be very wide: the width of the frequency range corresponds to «characteristic» (or «synchrotron») frequency $\omega_{sr} \approx \frac{2\pi}{T_{sr}}$. A wide range of wavelengths provides a lot of opportunities for using synchrotron radiation in radiography. The characteristic wavelength of a particular source is an important quantity for practical applications.

5) Determine the characteristic wavelength of the source described in 2). The answer must be the equation and the numerical value (in meters).

The main method of deciphering the structure of a crystal material is *X-ray diffraction*. The radiation is being diffracted (elastically scattered) by atoms of a sample. The crystal serves as a *diffraction grid* for X-ray beam because its wavelength is of the same order of magnitude as a spacing between atomic planes. When the radiation is incident on the crystal at some angle, the reflected radiation is detected not only in the direction determined be the laws of geometrical optics but also at the angles for which the waves reflected by adjacent planes have optical path difference equal to an integer of radiation wavelength. These reflected waves mutually amplify at a remote detector resulting in a significant rise of intensity in the corresponding direction (*diffraction maximum*).

6) Using the condition of diffraction maximum derive the explicit formula for the direction at a diffraction maximum of X-rays reflected by a crystal which lattice consists of a single set of parallel planes. The beam is incident at the angle θ to the planes, the interplane spacing equals d (see Fig.2).

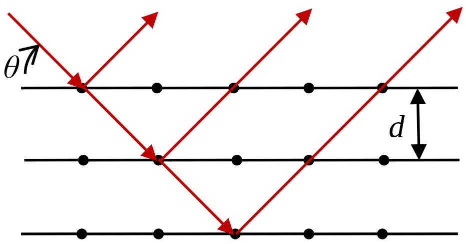


Fig. 2.

In a real crystal structure it possible to introduce different sets of equidistant parallel planes. Such a set can be defined by a vector perpendicular to the planes while directions at the diffraction maxima defined by the condition derived in 6) can be specified by diffraction angle 2θ (the angle between the incident and diffraction beams). In what follows the observed maximum of a given order for a specific set of parallel planes is called «*reflex*». Any atomic plane necessarily passes through the nodes of crystal lattice, so coordinates of the vector perpendicular to a particular set of parallel planes can be given by integers $\vec{K} = (h, k, l)$ providing the coordinate axes are aligned with the lattice principal axes (edges) and a distance is measured in lattice constants. Thus, a reflex can be determined by a set of three integers (hkl).

Superconductor is a very interesting object for the modern materials science. One of the most common and widely used low temperature superconductor is triniobium-tin Nb_3Sn. For instance, this superconductor is used in electrical circuits of the Large Hadron Collider. A unit cell of its lattice structure (i.e. a cell which repetitive translation along principal axes reproduces the whole crystal) is a cube of side L = 5,29 Å (1 Å = 10^{-8} cm).

7) Find the diffraction angle (the angle between the incident and diffracted beams) for reflex (110) at the first order of diffraction using the value of characteristic wavelength calculated above. The answer should be the formula and the numerical value (in radians).

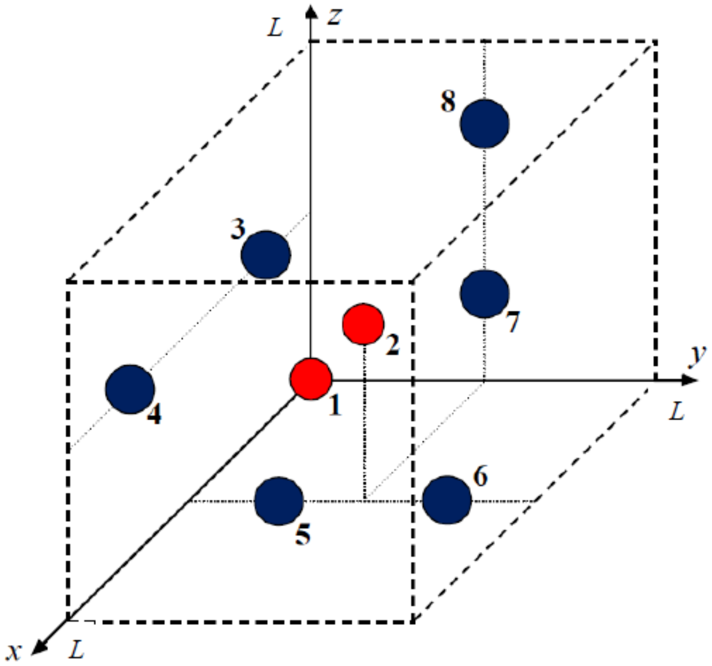


Fig. 3.

In the Cartesian frame, which coordinate axes are aligned with the edges of crystal lattice, the coordinates of Sn atoms (in the units of L) are: (0; 0; 0), (0,5; 0,5; 0,5), and the coordinates of Nb atoms are: (0,25; 0; 0,5), (0,75; 0; 0,5), (0,5; 0,25; 0), (0,5; 0,75; 0), (0; 0,5; 0,25), (0; 0,5; 0,75), (see Fig.3). Atoms in the unit cell are numbered from 1 to 8 in the order they are listed in the text. A pattern of X-ray scattering is determined by the distribution of electrons in a crystal lattice (electrons are so much lighter than atomic nuclei, so they respond much stronger to the electromagnetic field of incident wave), i.e. by distribution of atoms of various elements. The ability of an isolated atom of a certain element A to scatter radiation is specified by a quantity $f(A)$ called *atomic scattering factor*. This quantity specifies the difference of wave scattering by electronic shell of a given atom compared to that one by free electrons. Atomic scattering factor is a complex quantity (i.e. $f = \alpha + i\beta$, where $i \equiv \sqrt{-1}$ is imaginary unit) and its absolute value squared $|f|^2 = \alpha^2 + \beta^2$ determines intensity of scattered radiation of an isolated atom. The intensity of an observed diffraction peak for a crystal lattice is calculated as the squared absolute value of the *reflex structure factor* $I \approx |F|^2$, which in turn is evaluated as:

$$F = \sum_{n=1}^{8} a_n(A) \cdot f(A) \cdot e^{2\pi i(hx+ky+lz)},$$

where sum is over all atomic positions in the unit cell. Here (x, y, z) are coordinates of atom at the n-th position; $f(A)$ is the atomic scattering factor of element A which atom resides at the n-th position, and $a_n(A)$ is *occupation* of the position by the element. Occupation of a position is the average (over the whole lattice) number of atoms of a certain element at the position. In ideal crystal (i.e. when all atoms of crystal lattice reside at their nodes) each position is occupied exactly by a single atom of a given element and there are no «extra» atoms, i.e. an occupation is either 1 or 0. For example, in the unit cell of Nb_3Sn at the 1-st position: $a_1(\mathrm{Sn}) = 1$ and $a_1(\mathrm{Nb}) = 0$. However, a real crystal structure has defects distorting ideal lattice, so occupations can be different. One of the most common defect of this sort is the so-called *antinode disordering* when atoms switch their positions. For instance, if in some cells atoms of Sn at position 1 switch to position 3, and atoms Nb in these cells switch from 3 to 1, occupation $a_1(\mathrm{Sn})$ becomes less than 1 and $a_1(\mathrm{Nb})$ becomes non-zero. Nevertheless, the gross occupation of any position remains equal to 1, i.e. any atom leaving its position switches to position of another atom and vice versa.

8) Suppose there is an antinode disordering in the considered structure, so the occupation of Nb positions by atoms of Sn becomes equal to δ, i.e.

$$a_3(\mathrm{Sn}) = a_4(\mathrm{Sn}) = a_5(\mathrm{Sn}) = a_6(\mathrm{Sn}) = a_7(\mathrm{Sn}) = a_8(\mathrm{Sn}) = \delta.$$

Determine other occupations $a_n(A)$ in the structure. Occupations of atoms of any element at positions $n = 1$ and $n = 2$ are the same, as it is the case for positions $n = 3, \dots, 8$. Express the occupations via δ.

9) Assuming $f(\mathrm{Nb}) \equiv f_{\mathrm{I}}$ and $f(\mathrm{Sn}) \equiv f_{\mathrm{II}}$ to be known evaluate structure factor of reflex (110), using the occupations calculated in 8). The answer should be given as equation.

Hint: according to Euler's formula complex exponential is evaluated as $e^{i\varphi} = cos\varphi + i \cdot sin\varphi$.

10) Determine the condition of quenching the reflex (110) (when its intensity vanishes) for the same structure. The answer should be given as a numerical value of δ.

Problem 3: Neutrino

Neutrino is one of the most peculiar elementary particles. It has no electric charge and does not participate in the strong interactions (which are responsible for stability of atomic nuclei). Physicists use the word «flavor» to specify a neutrino type. There are three neutrino flavors known to date: electron neutrino ν_e, muon neutrino ν_μ, and tau-neutrino ν_τ. A neutrino of each flavor has its antiparticle (antineutrino). The symbols used for the latter are the same as for neutrinos but with an upper bar: $\bar{\nu}_e$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$.

Neutrino participates only in the weak interactions, the most famous process mediated by the weak interaction is β-decay. In this process a single neutron in atomic nucleus decays into a proton, an electron, and an electron antineutrino: $n \to p + e + \bar{\nu}_e$ (however, it would be a mistake to think than neutron is composed of these particles, there is also a process $p \to n + e^+ + \nu_e$!).

A neutrino is always created together with its antineutrino or a charged antilepton (positron e^+ (an electron antiparticle), antimuon μ^+, or antitau-lepton τ^+). An antineutrino, in turn, is always created together with its neutrino or the corresponding charged lepton.

Neutrinos are extremely lightweight, their masses are several orders of magnitude less than masses of other matter particles. The precise values of neutrino masses are still unknown. Due to their small masses all neutrinos participating in nuclear reactions are *ultrarelativistic*, i.e. their velocities are very close to the speed of light in vacuum. The energy of such a neutrino of mass m and momentum $\vec{p}$ is almost independent of its mass:

$$E = \sqrt{m^2c^4 + c^2\vec{p}^2} \approx c\,|\,\vec{p}\,|.$$

A neutrino, like many other elementary particles, has *spin*, i.e. the proper angular momentum, which is non-zero even in the neutrino rest frame. A specific feature of all detected neutrinos (antineutrinos) is the negative (positive) sign of its spin component projected on the direction of neutrino (antineutrino) momentum. Loosely speaking, a neutrino does not have a «mirror reflection». Other elementary matter particles can have both signs of the spin component. Physicists explain this fact by saying that neutrinos with other spin component either do not exist, or do not participate even in the weak interactions (so they cannot be detected).

Physical constants and data (can be used in any part of the problem)

- speed of light in vacuum $c \approx 3 \cdot 10^8$ m/s;
- gravitational constant $G \approx 6{,}7 \cdot 10^{-11}$ m^3/(kg·s^2);
- Planck constant $\hbar \approx 10^{-34}$ J·s;
- proton radius $r_p \approx 10^{-15}$ m;
- Avogadro constant $N_A \approx 6 \cdot 10^{23}$ mole^{-1};
- hydrogen molar mass $\mu \approx 2$ g/mole;
- Solar mass $M_C \approx 2 \cdot 10^{30}$ kg;
- Solar radius $r_C \approx 7 \cdot 10^8$ m;
- mean radius of Earth's orbit $a \approx 1{,}5 \cdot 10^{11}$ m;
- eccentricity of Earth's orbit $\varepsilon \approx 0{,}017$;
- radius of «active» solar core where nuclear fusion proceeds and neutrinos are created $r_a \approx 1{,}2 \cdot 10^8$ m;
- range of electron density n_e inside the Sun from the active core to outer layers: from $5{,}9 \cdot 10^{31}$ m^{-3} to 10^{29} m^{-3};
- parsec (pc), an astronomical unit of length, 1 pc $\approx 3{,}2$ light year $\approx 3 \cdot 10^{16}$ m.
- electronvolt (eV) is the unit of energy equal to the work done by electrostatic force moving a single electron across potential difference of 1 V.

Part I: neutrino masses and oscillations.

The Nobel Prize of 2015 was awarded for the «discovery of neutrino oscillations indicating that neutrinos are massive». Neutrino oscillations is a process of interconversion of neutrino flavors. According to modern theoretical models the possibility of neutrino oscillations is indeed closely related to their masses (massless neutrinos cannot oscillate).

It should be noted that neutrinos like other elementary particles are not some «immutable» entities, rather they are *quanta* of a neutrino field (similarly to photons which are quanta of electromagnetic field). Therefore, in different physical situations they can appear in the states with different properties. For instance, neutrino state of a *certain flavor* (a state in which neutrino is created or annihilated in nuclear reactions) does not coincide with neutrino state of a *certain mass*.

To be specific, consider oscillation $\nu_e \leftrightarrow \nu_\mu$ (i.e. we neglect the third neutrino flavor). An intensive flow of neutrinos can be regarded as «almost classical» radiation of a given wavelength (here the analogy with electromagnetic wave, an «almost classical» flow of a large number of photons, applies again).

The existence of several neutrino states can be described by introducing a «polarization»: $\vec{u}(t,\vec{r}) = \vec{u}_1 \cos(\omega_1 t - \vec{k}\vec{r}) + \vec{u}_2 \cos(\omega_2 t - \vec{k}\vec{r})$. The quotation mark indicates that this polarization is not a polarization in the «regular» space, this is polarization in the «space of neutrino states» although for our purposes this is almost insignificant. The flux of neutrinos is proportional to $\vec{u}^{\,2}$. Notice that the frequency ω and wavevector k of the wave are related to the energy and momentum of neutrinos by the common quantum formulae: $E = \hbar\omega$ and $\vec{p} = \hbar\vec{k}$, where $\hbar$ is *Planck constant*.

The difference of frequencies is due to difference in masses: for the same momentum $E_{1,2} = \sqrt{m_{1,2}^2 c^4 + c^2 \vec{p}^{\,2}} = \hbar\omega_{1,2}$. Obviously, an orthogonal «polarization» $\vec{u}_{1,2}$ corresponds to the neutrino state of certain mass $m_{1,2}$. Notice, that states of definite flavor (ν_e and ν_μ) correspond to another pair of orthogonal «polarizations» $\vec{u}_{e,\mu}$ **which do not coincide with** $\vec{u}_{1,2}$.

The polarizations $\vec{u}_{1,2}$ corresponding to certain masses and polarizations $\vec{u}_{e,\mu}$ corresponding to certain flavors are related as:

$$\begin{cases} \vec{u}_1 = \vec{u}_e \cos\vartheta - \vec{u}_\mu \sin\vartheta, \\ \vec{u}_2 = \vec{u}_e \sin\vartheta + \vec{u}_\mu \cos\vartheta. \end{cases}$$

Angle ϑ is called the «mixing angle» of ν_e and ν_μ. In this case ν_e and ν_μ indeed do not have «certain» masses and do not have a certain energy for a given momentum. For instance, a measurement of the energy of electron neutrino would «on average» yield the value $\langle E_e \rangle = E_1 \cos^2\vartheta + E_2 \sin^2\vartheta = E - \frac{\Delta E}{2}\cos 2\vartheta$, where $E \equiv \frac{E_1 + E_2}{2} \approx c\,|\vec{p}|$ is the mean neutrino energy and $\Delta E \equiv E_2 - E_1$. Such an outcome could be interpreted as being due to interaction of the states ν_e and ν_μ, where the interaction energy is $V_{e\mu} = \Delta E \sin\vartheta \cos\vartheta$.

The quantities introduced above can be expressed in terms of energy E and parameters $m \equiv \frac{m_1 + m_2}{2}$, $\Delta m \equiv m_2 - m_1$, and ϑ. It has been already mentioned that accurate values of these parameters are not known yet but to solve the problem it would suffice to adopt the following approximate values: $mc^2 = 4{,}0 \cdot 10^{-3}$ eV, $\Delta mc^2 = 3{,}0 \cdot 10^{-3}$ eV, and $\vartheta = 10°$.

1) Evaluate «mean» masses of electron and muon neutrinos for the given values of the parameters. The answer can be given either in kg or eV/c^2.

Now consider neutrinos radiated from some small region and propagating along x-axis. Let the energy of a neutrino created in this region be $E \approx 2$ MeV, all created neutrinos are electron neutrinos and have the same certain momentum (this means that neutrinos are created with different masses and, therefore, energies). Clearly, the neutrino wave propagating along *x*-axis is a mixture of neutrino waves corresponding to neutrinos of different masses (hence, a mixture of waves with certain frequencies at a given wavelength). The phase shift of the waves varies with distance *x*. Since the phase shift varies the contributions to the resulting wave due to electron and muon component would vary as well. Therefore, at any particular position *x* one would detect not only electron neutrinos but muon neutrinos as well. The intensities of the corresponding neutrino fluxes vary periodically in space. This phenomenon is called neutrino oscillations.

2) Determine the oscillation length, i.e. the period of spatial variation of the time averaged flux of muon neutrinos. The answer should be given as the formula and the numerical value (in meters).

Part II: neutrinos and the Sun.

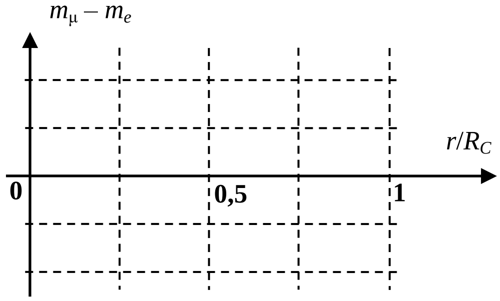
Oscillations described in Part I occur in vacuum. At first glance, it would be reasonable to assume that matter does not alter the picture significantly since neutrinos very weakly interact with any matter which density is much less than the density of atomic nucleus. There is a powerful neutrino source close to the Earth. It is the Sun. Nuclear reactions proceed in the central region of the Sun suppling it with energy and creating neutrinos and antineutrinos, mostly, electron ones. However, the observed flux of electron neutrinos turned out to be only a half of the flux predicted from the Solar luminosity. Is it possible to explain this «deficit» by partial conversion of electron neutrinos to neutrinos of other flavors on their way from the Sun to the Earth (vacuum oscillations)?

3) Try to give a justified answer by using the data and the results from Part I of the problem. Do necessary calculations to support your judgement. In particular, it is reasonable to assume that energies of neutrinos created in nuclear reactions in the Sun are not very different from 2 MeV. Take into account the fact that a neutrino detector accumulates data for a long period of time, up to 2-3 months. The answer should be given as «+» (yes) or «–» (no).

A detailed analysis must take into account that the neutrinos travel a part of their path inside the solar substance. It turns out, absorption of the neutrinos by the substance does not change much the estimate for the flux of electron neutrinos but there is some additional circumstance which is quite essential. The solar substance contains a lot of electrons (see the problem data) and electron neutrinos interact with them much stronger than muon neutrinos do. Due to this fact the «mean» energy of electron neutrinos increases by δE, the quantity which is proportional to the electron density: $\delta E \approx 1{,}27 \cdot 10^{-12} \dfrac{n_e}{10^{31}\ \mathrm{m}^{-3}}$ eV. At the same time the «mean» energy of muon neutrinos and the energy of interaction between the states ν_e and ν_μ **remain practically the same**.

4) Evaluate «new» value of mixing angle $\tilde{\vartheta}$ (by taking into account the solar electrons). The answer should be the equation.
5) By how many percent can the solar electrons change the expected «deficit» of the electron neutrino flux? Evaluate in percent (%) the maximum increment of the flux of muon neutrinos due to oscillations (compared to the flux in the absence of matter).

6) Plot an approximate dependence (i.e. show only the main features) of the mass difference of ν_μ and ν_e as a function of the distance r traveled inside the Sun from the center outwards.



Part III: neutrino and Supernova explosion.

After an «ordinary» star has exhausted its nuclear fuel the star cools down and its internal pressure cannot withstand gravitational compression anymore. As a result, the star is contracting until its substance undergoes transition to a new phase in which all electrons are «shared», all nuclei «float» in the electron «gas», and only the pressure of this gas halts further collapse.

The star can exist in this state for a long time; however, if the mass of its dense core gradually increases and exceeds $M_{\text{cr}} \approx 1{,}5 M_C$ the core becomes unstable and collapses. At the onset of collapse such a core usually has a radius about 10000 km with approximately equal numbers of protons and neutrons. Soon after the contraction starts the rate of electron-nuclear collisions becomes high, which results in *neutronization* of the star substance due to reaction of «inverse β-decay» ($p + e \rightarrow n + \nu_e$). Electron disappearance reduces the pressure of electron gas accelerating the neutronization even more. The whole process is essentially the tremendous explosion leaving in the aftermath a *neutron star* and an «outer envelope» flying outwards. Astronomers call such an explosion «Supernova explosion» or simply «Supernova». A neutron star is indeed composed mostly of tightly packed neutrons, so its density is approximately equal to the density of atomic nuclei.

7) Estimate an order of magnitude of the energy released due to compression of the stellar core from the initial radius to the neutron star. Calculate the numerical answer in Joules.

The released energy converts to kinetic energy of the star remnants (the flying outer layers and rotation of the neutron star) and to the energy of electromagnetic radiation and neutrinos. Supernova explosion is one of the most powerful source of neutrinos (sometimes it is called «neutrino bombs»), calculations show that more than half of the released energy converts to the energy of radiated neutrinos. Neutrinos are radiated both at the neutronization stage and after formation of the neutron star which is initially extremely hot and subsequently cools down mostly by radiating neutrinos. The neutronization and cooling take just several seconds. Notice that only electron neutrinos are created during the neutronization and neutrino-antineutrino pairs of various flavors are created during the cooling.

8) Estimate the number of neutrinos created by a Supernova explosion assuming that the mass of the initial stellar core is approximately $1{,}5M_C$ (the stellar substance does not «go away» with outer layers), the energy of radiated neutrinos is 80% of the released energy, and the mean energy of radiated neutrinos and antineutrinos is approximately 10 MeV. You could assume that neutron mass and radius are approximately the same as those of proton. Calculate the numerical values.
9) Supernova SN1987A exploded at the distance of $R \approx 50$ kpc from the Earth (in the Large Magellanic Cloud). What is the total number of neutrinos and antineutrinos passed through an Earth based detector of the cross-sectional area $S = 100$ m^2? Estimate the expected number of detected neutrinos and antineutrinos assuming that the detector on average registers $\alpha = 3 \cdot 10^{-14}$ % of neutrinos of any flavor in the corresponding energy range. Calculate the numerical values.

It is important that neutrino radiation is asymmetric with respect to the star magnetic axis due to «peculiar» neutrino behavior under mirror reflection: the power dI radiated in the infinitesimal solid angle $d\Omega = \sin\theta d\theta d\phi$ is $\frac{dI}{d\Omega} = \frac{I}{4\pi}[1 + \kappa \cdot \cos\theta]$, where $\kappa \approx 10^{-2}$, θ is the angle of neutrino emission to the axis, and ϕ is the angle of rotation around the axis.

10) Estimate the speed gained by the star due to neutronization and cooling. Calculate the numerical answer (in km/s).